Hierarchical quarks and anarchic neutrinos in warped Wpace

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I will review the generation of fermion masses and mixing angles for the quark and lepton sectors, in theories where the hierarchy problem is solved by a warped extra dimension.

The qualitative difference between both sectors will be outlined. In particular in the lepton sector the cases of Dirac and Majorana neutrinos will be covered independently.

In the latter case I will present a model where the PMNS angles can be fixed dynamically by the Wilson Lines of a bulk local flavor symmetry.
The outline of this talk is

Outline

- General motivation
- Warped dimensions (reminder)
- Hierarchical quark sector
  - EW constraints
  - FCNC & CPV
  - Combination of all constraints
- Anarchic neutrino sector
- Dirac neutrinos
- Majorana neutrinos
- Flavor from Wilson Lines
  - Gauge group
  - A model for $U_{PMNS}$
  - Lepton number violation
- Conclusion
LHC is trying to answer the fundamental question: What is the nature of the electroweak symmetry breaking? Is it a perturbative mechanism [as the Brout-Englert-Higgs (BEH) one] or a non-perturbative one [as in QCD]?

From the experimental point of view, everything seems to be consistent with the SM with the BEH mechanism [aka Higgs mechanism] and possibly with a Higgs scalar around 125-126 GeV. However, from the theoretical point of view, such electroweak vacuum is not stable under quantum corrections [aka hierarchy problem] and new physics should stabilize it.

Uncovering the nature of electroweak breaking should amount to uncovering the kind of new physics (if any) which stabilizes the electroweak vacuum.
There are two main avenues for solving the hierarchy problem

**Elementary Higgs**
- There should exist an **extra symmetry** and **new particles** with couplings dictated by this symmetry such that quadratic sensitivity to high scale cancels.
- Typical example is **supersymmetry**: stops cancel the quadratic divergence generated by the top quark.

Not dealing with in today’s talk.

**Composite Higgs**
- At some scale the Higgs **dissolves** and the theory of constituents is at work.
- Similar to QCD where the pions dissolve into quarks.
- The **compositeness scale** acts as a cutoff of quadratic divergences.
- Typical example of compositeness is **technicolor**.
- Modern theories of compositeness involve an **extra dimension** through the AdS/CFT correspondence.
The original AdS/CFT correspondence relates 5D theories of gravity in AdS to 4D strongly-coupled conformal field theories. In the case of a slice of AdS a similar correspondence can also be formulated.

Boundary at $y = 0$ corresponds to UV cutoff in the 4D CFT. $y = y_1$ corresponds to IR cutoff. Matter at UV is elementary: e.g. light fermions. Matter at IR is composite: e.g. KK modes, heavy fermions, ...

Although the CFT picture is useful for understanding some qualitative aspects of the theory it is useless for obtaining quantitative predictions since the theory is strongly coupled.
An AdS 5D theory with two branes was proposed long ago \(^1\)

\[
ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2
\]

To solve the hierarchy problem the Higgs should be either

- Localized on the IR brane (composite): theory is EWPT disfavored
- Propagating in the bulk but with a profile along the extra dimensions leaning towards the IR brane (a degree of compositeness)

\[
h(y) \sim e^{aky} \Rightarrow a > 2 \text{ (to solve hierarchy problem)}
\]

The hierarchy problem is solved because the Planckian Higgs mass is warped down to the weak scale by the geometry or in the dual picture because the conformal operator breaking EW has dimension \(> 2\)

To ”solve” the flavour problem fermions should propagate in the bulk and with different localizations along the extra dimension

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\(^1\) L. Randall and R. Sundrum, hep-ph/9905221
Hierarchical quark sector

Hierarchical quark sector

Quark (fermion) localization is controlled by the 5D Dirac mass term

\[
\mathcal{L}_m = - \int dy e^{-4ky} c_\psi k \bar{\psi}_R \psi_L + h.c. \quad c_\psi = (-c_L, c_R)
\]

\[
\psi(x, y) \simeq f_\psi^{(0)}(y) \psi^{(0)}(x) + \cdots, \quad f_\psi^{(0)}(y) \simeq e^{(1/2 - c_\psi)ky}
\]

Light fermions

\( c > \frac{1}{2} \quad \text{fermion Higgs} \)

Heavy fermions

\( c < \frac{1}{2} \quad \text{Higgs fermion} \)
The structure of Yukawa couplings is \( \epsilon = e^{-ky_1} \simeq m_{KK}/k \)

\[
Y_{ij} \simeq \hat{Y}_{ij} f(c_L^i) f(c_R^j), \quad f(c) = \sqrt{|2c - 1|} \left\{ \begin{array}{ll}
1 & (c < 1/2) \\
\epsilon^{c-1/2} & (c > 1/2)
\end{array} \right.
\]

So the Yukawa matrices are \( x \equiv c_x - 1/2, \ x = q_i, u_i, d_i \)
\( (q_1 > q_2 > q_3, \ u_1 > u_2 > u_3, \ d_1 > d_2 > d_3) \)

\[
Y_{ij}^U \sim \begin{pmatrix}
\epsilon^{q_1+u_1} & \epsilon^{q_1+u_2} & \epsilon^{q_1} \\
\epsilon^{q_2+u_1} & \epsilon^{q_2+u_2} & \epsilon^{q_2} \\
\epsilon^{u_1} & \epsilon^{u_2} & 1
\end{pmatrix}, \quad Y_{ij}^D \sim \begin{pmatrix}
\epsilon^{q_1+d_1} & \epsilon^{q_1+d_2} & \epsilon^{q_1+d_3} \\
\epsilon^{q_2+d_1} & \epsilon^{q_2+d_2} & \epsilon^{q_2+d_3} \\
\epsilon^{d_1} & \epsilon^{d_2} & \epsilon^{d_3}
\end{pmatrix}
\]

Typical of hierarchical mixing angles and mass eigenvalues

\[
s_{ij} \sim \epsilon^{q_i-q_j}, \ (i < j), \ m_i \sim \nu \epsilon^{q_i}, \ q_3 \equiv 0
\]
This qualitative behavior is consistent with the hierarchical behavior of quark masses and mixing angles in CKM matrix:

\[ s_{12} \approx 0.226, \quad s_{23} \approx 0.042, \quad s_{13} \approx 0.004 \]

\[
\text{mass}/m_t : (u, c, d, s, b) \approx (2 \cdot 10^{-5}, 7 \cdot 10^{-3}, 3 \cdot 10^{-5}, 6 \cdot 10^{-4}, 2 \cdot 10^{-2})
\]

- We have performed a $\chi^2$ fit to the experimental quark masses and CKM matrix elements.²
- We have randomly generated a set of 40,000 complex 5D Yukawas and fitted the 9 parameters $c_\psi$.

The result of the fit (including $1\sigma$):

<table>
<thead>
<tr>
<th></th>
<th>$c_{(u,d)}^L$</th>
<th>$c_{(c,s)}^L$</th>
<th>$c_{(t,b)}^L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{uR}$</td>
<td>0.71 ± 0.02</td>
<td>0.57 ± 0.02</td>
<td>-0.11 ± 0.45</td>
</tr>
<tr>
<td>$c_{dR}$</td>
<td>0.66 ± 0.03</td>
<td>0.65 ± 0.03</td>
<td>0.42 ± 0.05</td>
</tr>
<tr>
<td>$c_{cR}$</td>
<td>0.57 ± 0.02</td>
<td>0.65 ± 0.03</td>
<td></td>
</tr>
<tr>
<td>$c_{sR}$</td>
<td>0.65 ± 0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{bR}$</td>
<td>0.64 ± 0.02</td>
<td></td>
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</tr>
</tbody>
</table>

The probability distribution functions: dashed lines
The model has to be contrasted with electroweak constraints, in particular with the universal "oblique" and non-universal ones, leading to lower bounds on $m_{KK}$.
The RS model has to be modified to avoid the large corrections to EW observables. It can be done in two ways

- By introducing a gauge custodial symmetry in the bulk $SU(2)_R^3$ which relieves tension with $T$
- By deforming the AdS metric in the IR which localizes more the KK modes on the IR brane reducing overlapping with the light fermions

A particularly simple model for the metric has been used as

### IR deformed model

$$A(y) = ky - \frac{1}{\nu^2} \log \left(1 - \frac{y}{y_s}\right)$$

$$\phi(y) = -\frac{\sqrt{6}}{\nu} \log[\nu^2 k (y_s - y)]$$

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The deformed model behaves much better than RS for EW observables. Consider e.g.

\[ k\Delta = 1, \; \nu = 1/2 \]
Models with different fermion localization generate FCNC and CPV by exchange of KK bosons and produce $d=6$ effective operators.

The main constraints come from

$$C_4^{sd}(\bar{d}_L s_R)(\bar{d}_R s_L), \quad \Delta S = 2, \quad \text{Im} \ C_4^{sd} \leftrightarrow \epsilon_K$$
ALL CONSTRAINTS

Combining all possible constraints one gets

PDF: solid = deformed

CDF

<table>
<thead>
<tr>
<th>Probability for $m_{KK}$ below</th>
<th>10%</th>
<th>20%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 TeV</td>
<td>5 TeV</td>
<td>10 TeV</td>
<td></td>
</tr>
<tr>
<td>RS</td>
<td>13 TeV</td>
<td>16 TeV</td>
<td>21 TeV</td>
</tr>
<tr>
<td>$\nu = 0.5$</td>
<td>3.3 TeV</td>
<td>4.2 TeV</td>
<td>7.2 TeV</td>
</tr>
</tbody>
</table>
**ANARCHIC NEUTRINO SECTOR**

- While the charged lepton sector spectrum is hierarchical

\[ \text{mass}^2 / m_T^2 : (e, \mu) \simeq (9 \cdot 10^{-8}, 4 \cdot 10^{-3}) \]

- and can be described by a “quark-like” formalism where \( c_\varepsilon_1 > c_\varepsilon_2 > c_\varepsilon_3 \)

- The neutrino sector is *anarchic* \(^5\) which means that masses are not so different and mixing angles (in the PMNS matrix) are \( O(1) \)

\[ s_{12} \simeq 0.559, \quad s_{23} \simeq 0.721, \quad s_{13} \simeq 0.152 \]

\[ \Delta m^2_{\odot} / \Delta m^2_A \simeq 3 \cdot 10^{-2} \]

- and suggests a non-hierarchical scenario where

\[ \text{SYMMETRY } \Rightarrow \quad c_{\ell i} \simeq c_{\ell}, \quad c_{N i} \simeq c_{N}, \quad \forall i \]

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**Dirac neutrinos**

- If lepton number is not violated either in the bulk or in the branes neutrinos are Dirac fermions
- For 5D RH neutrinos one can introduce a 5D Dirac $c_N$ mass

$$\mathcal{L}_{\text{Dirac}} = \int e^{-4ky} k \left( -\bar{N}_R c_N N_L + \text{h.c.} \right)$$

- A summary of results for neutrino mass matrix (assuming a bulk symmetry $\Rightarrow c_{\ell i} \simeq c_{\ell}, \ c_{N i} \simeq c_N, \ \forall i$)

**The case of Dirac neutrinos**

- For bulk and/or IR localized Yukawa coupling

$$m_{ij} \simeq O(1) \nu \epsilon^{c_{\ell}^{-1/2}} \epsilon^{c_N^{-1/2}} \gtrsim O(1) \nu m_\tau \epsilon^{c_N^{-1/2}} \text{ allowed for } c_N \gtrsim 5/6$$

- For UV Yukawa coupling: $m_{ij} \sim O(1) \nu \epsilon^{a-1}$ excluded (too small)
For 5D RH neutrinos one can introduce a Dirac $c_N$ AND a Majorana mass matrix (in the bulk $c_M$ and on the branes $n_{0,1}$)

\[
\mathcal{L}_{\text{mass}} = \int e^{-4ky} k \left( -\bar{N}_R c_N N_L + \text{h.c.} + \frac{1}{2} [N_L c_M N_L - N_R c_M N_R + \text{h.c.}] \right)
\]

\[
\mathcal{L}_{bd} = \left[ \frac{1}{2} \bar{N}_R n_0 \bar{N}_R + \text{h.c.} \right]_{y=0} - \left[ e^{-4ky_1} \frac{1}{2} \bar{N}_R n_1 \bar{N}_R + \text{h.c.} \right]_{y=y_1}
\]

We compute the 5D propagator at zero momentum \(^6\)

\[
G^{ab}_{RR}(y, y') = \sum_n \frac{f_{N_R}^{(n)}(y) f_{N_R}^{(n)}(y')}{m_n}
\]

Integrating out (the whole tower of) RH neutrinos gives rise to the 4D neutrino operator

\[ \mathcal{L}_W = c_W^{ij} \left[ \bar{\ell}_i^{(0)}(x) \cdot \tilde{H}^{(0)}(x) \right] \left[ \tilde{H}^T(0)(x) \cdot \ell_j^{c(0)}(x) \right] + \text{h.c.} \]

and the neutrino mass matrix

\[ m_{\nu}^{ij} = c_W^{ij} \nu^2. \]
A summary of results for neutrino mass matrix (assuming a bulk symmetry ⇒ $c_{\ell i} \simeq c_{\ell}, c_{\mathcal{N} i} \simeq c_{\mathcal{N}}, \forall i$) follows:

### The case $c_M \neq 0$: violation of lepton number in the bulk

- For bulk and/or IR localized Yukawa coupling

  $$m_{ij} \sim O(1)^{ij} \frac{v^2}{\epsilon k} \epsilon^{2c_{\ell} - 1} \geq O(1)^{ij} \frac{m^2_{\tau}}{\epsilon k}$$

  excluded (too large)

- For UV Yukawa coupling:

  $$m_{ij} \sim O(1)^{ij} \frac{v^2}{\epsilon k} \epsilon^{2a - 1}$$

  excluded (too small)

### The case $c_M = 0$ (but $n_i \neq 0$): lepton number conservation in the bulk

- For bulk and/or IR localized Yukawa coupling

  $$m_{ij} \sim O(1)^{ij} \frac{v^2}{\epsilon k} \epsilon^{2c_{\ell} - 1} \cdot \epsilon^{c_{\mathcal{N}} (n_0)^{-1} c_{\mathcal{N}}}, \text{ allowed}$$

- For UV Yukawa coupling:

  $$m_{ij} \sim O(1)^{ij} \frac{v^2}{\epsilon k} \epsilon^{2a - 1}$$

  excluded (too small)
Naturalness & anarchic neutrino structure requires a bulk symmetry.

The continuous bulk symmetry has to be local not to be spoiled by quantum gravity effects (widely believed by stringy people).

Consider a bulk group $G \to H_0 \,(H_1)$ at the UV (IR) boundary.

The theory has zero modes for $A_\mu \in H_0 \cap H_1 \equiv H$ and $A_5 \in K_0 \cap K_1 \equiv K$ with $K_i \equiv G/H_i$.

One can transform away $A_5$ by a gauge transformation which changes the UV BC's of all fields.

$$\Lambda(y) = i \int_{y}^{y_1} A_5(y), \quad \psi(0) \to e^{i\Lambda(0)} \psi(0)$$

The UV boundary condition for the RH neutrinos changes as

$$n_0 \to e^{-i\Lambda(0)} n_0 e^{-i\Lambda(0)^T}$$
Various authors have built models that reduce LFV via
- A continuous symmetry \(^7\)
- A discrete symmetry \(^8\)

**In choosing \(G\) one should take into account the following requirements:**
- \(G\) needs to be large enough such that the breaking \(G \rightarrow H\) allows for nontrivial Wilson lines.
- In general, the larger \(G\), the more the theory will be protected from FCNC
- \(G\) should ensure degeneracy of the \(c_{\ell}\), but allow non-degenerate \(c_{\xi}\)
- \(G\) should not be so large such that the breaking \(G \rightarrow H\) leaves over unwanted zero modes for \(A_{\mu}\) (4D gauge symmetries)

The simplest bulk and boundary gauge groups:

- **Bulk group:**
  \[
  G = U(3)_{\ell} \otimes U(3)_{\mathcal{N}} \otimes i U(1)_{\mathcal{E}i} \{\lambda_{\mathcal{E}}^{3}, \lambda_{\mathcal{E}}^{8}, \lambda_{\mathcal{E}}^{0}\}
  \]
  \[
  \Downarrow
  \]
  \[
  Y_{\mathcal{N}}^{B} = Y_{\mathcal{E}}^{B} = c_{M} = 0, \quad c_{\ell}, \ c_{\mathcal{N}}, \ c_{\mathcal{E}i}
  \]

- **IR group:**
  \[
  H_{1} = \otimes_{i} U(1)_{(\ell+\mathcal{E}+\mathcal{N})i} = \{\lambda_{\mathcal{E}}^{3} + \lambda_{\ell}^{3} + \lambda_{\mathcal{N}}^{3}, \ \lambda_{\mathcal{E}}^{8} + \lambda_{\ell}^{8} + \lambda_{\mathcal{N}}^{8}, \ \lambda_{\mathcal{E}}^{0} + \lambda_{\ell}^{0} + \lambda_{\mathcal{N}}^{0}\}
  \]
  \[
  \Downarrow
  \]
  \[
  n_{1} = 0, \quad Y_{\mathcal{E}ij}^{1} = Y_{\mathcal{E}i}^{1} \delta_{ij}, \quad Y_{\mathcal{N}ij}^{1} = Y_{\mathcal{N}i}^{1} \delta_{ij}, \quad \Rightarrow \quad Y_{\mathcal{N}}^{1} = Y \propto \text{diag}(y_{1}, y_{2}, 1)
  \]

- **UV group:**
  \[
  H_{0} = U(3)_{\ell} \otimes U(1)_{\mathcal{N}^{1}} \otimes i U(1)_{\mathcal{E}i}
  \]
  \[
  \Downarrow
  \]
  \[
  Y_{\mathcal{N}}^{0} = Y_{\mathcal{E}}^{0} = 0, \quad n_{0} \lambda_{\mathcal{N}}^{1} + (\lambda_{\mathcal{N}}^{1})^{T} n_{0} = 0 \Rightarrow n_{0}^{-1} \propto \text{diag} \left(\frac{\nu_{3}}{2}, -\frac{\nu_{3}}{2}, 1\right)
  \]
A quick glance at Yukawas shows that they belong to the case where neutrino mass matrix can be accommodated.

One can easily check that

\[ H = H_0 \cap H_1 = \emptyset, \quad K = \left\{ \lambda^2, 4, 5, 6, 7 \right\} \supset \left\{ \lambda^2, 5, 7 \right\} = SO(3)_{\mathcal{N}} \]

And the mass matrix is

\[ m_{\nu}(b_k, y_k) \propto \text{diag}(y_1, y_2, 1) \cdot \hat{U}^T(b_k) \cdot \text{diag} \left( \frac{y_3}{2}, -\frac{y_3}{2}, 1 \right) \cdot \hat{U}(b_k) \cdot \text{diag}(y_1, 1, y_2) \]

with

\[ \Lambda_0(b_k) = \begin{pmatrix} 0 & -b_3i & b_2i \\ b_3i & 0 & -b_1i \\ -b_2i & b_1i & 0 \end{pmatrix}, \quad \hat{U}(b_k) \equiv e^{i\Lambda_0(b_k)} \]
- There is a qualitative difference between the parameters $b_k$ and $y_k$
  - Parameters $b_k$ should be obtained dynamically from CW potential
  - The Yukawas $y_k$ are free (anarchic) $O(1)$ parameters
- Here we will make a $\chi^2$ fit to mass eigenvalues and neutrino mixing angles
- As it is clear that there is room for the fit we will consider some examples first for fixed values of $y_k$ and then $b_k$

$y_k = (0.90, 0.95, 0.90)$ with best fit $@ \chi^2_{min} \simeq 0$ for $b_k^0 = (0.84, 0.11, 0.62)$ and $m_i \simeq (0.022, 0.024, 0.055)$ eV: direct hierarchy spectrum
Flavor from Wilson Lines

A WL model for $U_{PMNS}$

$b_k = 0.7$, $m_i \approx (0.004, 0.010, 0.050)$ eV, $\chi^2_{min} \approx 1.3$: direct hierarchy

$b_k = 0.4$, $m_i \approx (0.057, 0.058, 0.031)$ eV, $\chi^2_{min} \approx 2$: inverted hierarchy
LEPTON FLAVOR VIOLATION

- $c_\mathcal{E}, c_L$ and $Y_\mathcal{E}$ are diagonal
- *All charged lepton* couplings to electroweak KK gauge bosons preserve flavor, and there are no tree-level mediated FCNC’s as

$$\mu \rightarrow 3e, \mu - e \text{ conversion}$$

- At one-loop the leading diagram contributing to $\mu \rightarrow e\gamma$

$$Y_{\mathcal{N}} \rightarrow \hat{U} Y_{\mathcal{N}}$$

$$m_{KK} \gtrsim 2 - 3 \text{ TeV}$$

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CONCLUSION

A realistic spectrum and mixing angle pattern for neutrinos requires

- A bulk symmetry implementing that \( c^i_\ell \equiv c_\ell \) and \( c^j_N \equiv c_N \) independently on \( i, j \)
- A Yukawa matrix \( Y_N \) with non-vanishing components along the bulk and/or the IR brane. A UV localized Yukawa matrix alone would provide too small neutrino masses
- Lepton number should not be violated in the bulk. Otherwise the charged lepton spectrum would lead to a too heavy neutrino spectrum. Lepton number violating effects are thus dominated by those from the UV brane

We have constructed a simple model leading to the above required pattern for 5D masses, Yukawa couplings and lepton number violation
The Majorana mass matrix will then depend on the WL and leads to nontrivial mixing.

A priori the background \( \langle A_5 \rangle \) is a flat direction at tree-level (a classical modulus) which will however be dynamically determined at one-loop by the Coleman-Weinberg effective potential (the Hosotani mechanism). This will then result in a dynamical determination of the Majorana neutrino mass matrix.

We expect that moduli (flavons) get a mass

\[
m^2 \sim \left( \frac{g_F^2}{16\pi^2} \right) m_{KK}^2
\]

Computing the one-loop radiative corrections is to a large extent model dependent and it is a future task.

One might extend these ideas to the quark sector as \( c_d^i \equiv c_d \, \forall i \): and improve naturalness and flavor violation.

We have worked out RS metric but other metrics (e.g. soft-wall like) can be easily worked out.