Lifshitz Geometries in String and M-Theory

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AdS/CFT

The AdS/CFT correspondence is one of the most important developments in string/M-theory. In its simplest form it says:

String or M-Theory propagating on an $AdS_{d+1} \times M$ background

$\uparrow$

A quantum field theory in d spacetime dimensions that is conformally invariant.

Extraordinary that quantum gravity can be just QFT!
Extraordinary that certain QFTs are in fact quantum gravity!
Holography

Anti-de-Sitter space in d+1 spacetime dimensions

Conformal Field Theory in d spacetime dimensions
• Bulk Fields $\longleftrightarrow$ Operators in CFT e.g.

\[ g_{\mu\nu} \longleftrightarrow T_{ab} \]
\[ A_\mu \longleftrightarrow J_a \]

• Quantities in CFT obtained by considering $Z_{String}$ with fields having specific boundary conditions at infinity

• In general the CFTs that appear are gauge theories with $N$ degrees of freedom

• Supergravity approximation of string/M-theory is valid for the CFT at large $N$ and strong coupling.
• The AdS/CFT correspondence can be used to study the CFT at finite temperature $T$ by studying black holes

\[
T_H = \text{Temperature of the CFT} \\
S_{BH} = \text{Entropy of the CFT}
\]
• The AdS/CFT correspondence can be generalised to equivalences between non-conformally invariant quantum field theories (QFTs) and strings/M-theory propagating on other bulk spacetime backgrounds in several ways. We will see some examples soon.

• In all cases it is a strongly coupled limit of the QFT that can be described by a weakly coupled supergravity description.

Supergravity is a powerful tool to study infinite classes of strongly coupled quantum field theories.
Can we use the AdS/CFT correspondence to study strongly coupled condensed matter systems?

- One focus: systems with strongly coupled “quantum critical points” - phase transitions at zero temperature.

- Another focus: superconductors (superfluids) [Gubser; Hartnoll, Herzog, Horowitz].

  In fact some superconductors (“heavy fermions”, high Tc cuprates) are associated with quantum critical points.
• If a critical point has full relativistic conformal invariance in the far IR, one can aim to find AdS solutions of string or M-theory that describe the system.

• The dual description of the CM system at finite temperature is given by asymptotically AdS black hole solutions.

• One can study the CM system at finite chemical potential (or finite charge density) by constructing electrically charged AdS black holes.

• Leads to novel black hole solutions with interesting thermodynamic instabilities and an interesting array of phase transitions

• e.g. holographic superconductivity is described by electrically charge black holes with charged hair that spontaneously break the U(1) symmetry.
• We will study precisely such a setup later.

• However, it is not necessary that the quantum critical point is conformally invariant. In particular it is possible that it exhibits an anisotropic scaling.

First recall the metric for $AdS_{d+1}$ in Poincare coordinates

$$ds^2 = r^2(-dt^2 + dx^i dx^i) + \frac{dr^2}{r^2}$$

with $i = 1, \ldots, d - 1$. Observe that it is invariant under the isotropic scaling

$$t \rightarrow \lambda t, \quad x^i \rightarrow \lambda x^i$$

provided that we also scale

$$r \rightarrow \lambda^{-1} r$$
• The CM critical points may exhibit a scaling that is not isotropic (unlike a CFT)

\[ t \rightarrow \lambda^z t, \quad x^i \rightarrow \lambda x^i, \quad z \neq 1 \]

where \( z \) is the “dynamical exponent”. Note: \( i = 1, \ldots, d - 1 \)

• \textbf{Lifshitz}(\( z \)) geometries \cite{Kachru, Liu, Mulligan}

\[
\begin{align*}
ds^2 &= -r^{2z} dt^2 + r^2 (dx^i dx^i) + \frac{dr^2}{r^2} \\
\end{align*}
\]

\[ t \rightarrow \lambda^z t, \quad x^i \rightarrow \lambda x^i, \quad r \rightarrow \lambda^{-1} r \]

Call these \( \text{Lif}_{d+1}(z) \) geometries and so \( \text{Lif}_D(z = 1) = \text{AdS}_D \)
• Schrodinger\((z)\) geometries \([\text{Son; Balasubramanian, McGreevy}]\)

\[
\begin{align*}
\text{ds}^2 &= -r^{2z} \text{dt}^2 + r^2 \left(2d\xi \text{dt} + dx^i dx^i\right) + \frac{dr^2}{r^2} \\
\text{t} &\rightarrow \lambda^z \text{t}, \quad x^i \rightarrow \lambda x^i, \quad r \rightarrow \lambda^{-1} r, \quad \xi \rightarrow \lambda^{2-z} \xi
\end{align*}
\]

These geometries have an extra holographic coordinate, \(\xi\)

\[
\frac{\partial}{\partial \xi} \quad \text{dual to particle number}
\]

\[
-t \partial_i + x^i \partial_\xi \quad \text{dual to non relativistic boosts}
\]

When \(z=2\) there are also special conformal transformations - Schrodinger algebra

For both classes we also would like to construct black holes that asymptote to these geometries, as described before.

For these cases holographic dictionary not yet as well understood!
Most work has been carried out in “Bottom Up” models. Find solutions in a simple theory of gravity with a few other degrees of freedom e.g. a vector, plus one or two scalar fields.

Advantages:
• Simple
• Models should exist somewhere in string landscape?
• Might capture some universal behaviour?

Disadvantages:
• Does the model arise in string theory? Is there any well defined dual (conformal) field theory?
• If phenomenological model is viewed just as an approximation to a model that can be embedded in string theory, it might not capture e.g. interesting low temperature behaviour.
Alternative “Top Down” approach - construct explicit solutions of D=10 or D=11 supergravity (also brane probe limits)

Advantages:
• One is studying bone-fide dual field theories
• One can often study infinite classes of solutions - universality
• Can find new phenomenon (eg superconducting domes [JPG, Sonner, Wiseman])

Disadvantages:
• Hard! and harder to find solutions relevant for physical systems?

We have been pursuing Top Down supergravity constructions.

One important tool: Consistent Kaluza-Klein truncations
Schrodinger(z) Solutions

First examples were Bottom Up.

First Top Down constructions used either duality transformations and/or consistent KK truncations. [Maldacena, Martelli, Tachikawa] [Adams, Balasubramanian, McGreevy] [Herzog, Rangamani, Ross]

Led to infinite examples of (supersymmetric) solutions: [Yoshida, Hartnoll] [Donos, JPG]

**Type IIB:** uses $D=5$ (Sasaki-)Einstein spaces to construct solutions dual to $d=3$ field theories with Sch(z) symmetry with $z \geq 3/2$

**D=11:** uses $D=7$ (Sasaki-)Einstein spaces to construct solutions dual to $d=2$ field theories with Sch(z) symmetry with $z \geq 5/4$
Lifshitz(z) Solutions

Bottom Up: examples are well studied

Top Down: Can they be realised in String/M-theory?

Li, Nishioka, Takayangi: No-go theorem for SUGRA solutions
Hartnoll, Polchinski, Silverstein, Tong: Three schematic constructions.
Balasubramanian, Narayan: Recent examples with $z=2$

New results:
Donos, JPG using Sasaki-Einstein spaces:

Infinite $Lif_4(z = 2)$ solutions of Type IIB and D=11 SUGRA

Infinite $Lif_3(z = 2)$ solutions of D=11 SUGRA

Donos, JPG, Kim, Varela:
Single $Lif_4(z \sim 39)$ solution of D=11 SUGRA
Plan

- Consistent KK truncations
- A new consistent KK truncation of D=11 SUGRA on $\Sigma_3 \times S^4$ where $\Sigma_3 = H^3, S^3, R^3$
- Construction of $\text{Lif}_4(z \sim 39)$ solution of D=11 SUGRA
- Other AdS/CMT applications of new KK truncation: interesting new black holes
Consistent KK Truncations

• Consider KK reduction of some high D dimensional theory on some internal manifold M. Obtain a low d dimensional theory with an infinite number of fields.

• A consistent KK truncation is one where we can keep a finite set of fields such that any solution of the low d theory involving these fields uplifts to an exact solution of the high D theory.

• E.g. KK reduction on $S^1$ - keep ALL $U(1)$ invariant modes, the usual $g, A_\mu, \phi$. The modes which are truncated are charged and can’t source these neutral modes.

• Note that in this case it so happens that the modes that are kept are massless and the ones that are discarded are massive, so one can also view the truncation in an effective field theory sense.

• Usually consistent KK truncations don’t exist.
Some general results known in context of AdS/CFT

**Conjecture-Theorem (in many cases):**
Consider most general supersymmetric $\text{AdS} \times M$ solutions of D=10/11 SUGRA.
Can always consistently KK reduce on $M$ keeping the supermultiplet containing the graviton.
In the dual CFT language keep the fields dual to the superconformal current multiplet $(T_{ab}, J_a, \ldots)$

[JPG,Varela]

For $\text{AdS}_4 \times \text{SE}_7$ solutions of D=11 SUGRA and $\text{AdS}_5 \times \text{SE}_5$ solutions of type IIB SUGRA we can do a lot more by keeping breathing mode multiplets.

[JPG, Kim,Varela,Waldram][Cassani,Dall’agata,Faedo][Liu,Szepiotowski,Zhao]
[JPG, Varela][Skenderis,Taylor,Tsimpis]

Have been used to study top-down holographic superconductivity

[JPG,Sonner,Wiseman][Gubser,Herzog,Pufu,Tesileanu]
Consistent Truncation from Wrapped M5-branes and Lifshitz(z) Solutions of D=11 SUGRA

- Calabi-Yau $(M_6, J, \Omega)$ with a SLag 3-cycle $\Sigma_3$:

$$\text{Vol}(\Sigma_3) = \text{Re}(\Omega)|_{\Sigma_3}$$

- An M5-brane can wrap $\Sigma_3$ and preserve supersymmetry.

- Worldvolume of M5 is $\mathbb{R}^{1,2} \times \Sigma_3$. It preserves supersymmetry because R-symmetry currents are switched on. In the IR we obtain a d=3 QFT with N=2 susy.

- Via AdS/CFT we know that if $\Sigma_3 = H_3/\Gamma$ then this QFT is actually an N=2 SCFT that is dual to an $AdS_4 \times H^3/\Gamma \times S^4$ solution of D=11.
• Construct D=11 solutions that asymptote to \( AdS_7 \times S^4 \)
in the UV with

\[
ds^2(AdS_7) = \frac{dr^2}{r^2} + r^2 \left[ dx^\mu dx^\mu + ds^2(H^3/\Gamma) \right]
\]

and in the IR to \( AdS_4 \times H^3/\Gamma \times S^4 \)

This describes an RG flow “across dimensions” to the N=2 SCFT in d=3

Note: the metric is a warped product and the \( S^4 \) is fibred over \( H^3/\Gamma \)

Programme: study this N=2 SCFT at finite temperature and charge density
(with respect to the abelian R-symmetry) using a consistent KK truncation of D=11 SUGRA.
There exists a consistent KK truncation of D=11 SUGRA on \( \Sigma_3 \times S^4 \) where \( \Sigma_3 = H^3/\Gamma, S^3, T^3 \) Donos,JPG,Kim,Varela

Step 1: Reduce D=11 SUGRA on \( S^4 \) to get D=7 SO(5) gauged SUGRA.

Step 2: Reduce D=7 SO(5) gauged SUGRA on \( \Sigma_3 \) keep breathing mode multiplet

Obtain:

D=4 N=2 gauged supergravity (metric, vector)
+1 Vector multiplet (1 vector + 2 scalars)
+2 Hypermultiplets (8 scalars)

Scalar parametrise the coset \( \frac{SU(1,1)}{U(1)} \times \frac{G_2(2)}{SO(4)} \)
When $\Sigma_3 = H_3/\Gamma$ this D=4 theory has a susy $AdS_4$ solution which uplifts to the $AdS_4 \times H_3/\Gamma \times S^4$ solution.

This D=4 theory has a susy $\text{Lif}_4(z)$ solution which uplifts to $\text{Lif}_4(z) \times H^3/\Gamma \times S^4$ with $z \sim 39$

This avoids no-go theorem due to fibration structure.
Can also use the D=4 truncated theory to study the N=2 SCFT dual to M5-branes wrapping SLag $H^3/\Gamma$ cycles at finite $T$ and $\mu$

The usual D=4 AdS-RN black hole (brane) of Einstein-Maxwell theory describes the SCFT at high temperatures. Are there additional branches of black hole solutions as one lowers the temperature?

Study (some) linearised fluctuations within the D=4 theory. Find two new branches of black hole solutions:

1. Branch of black holes carrying charged scalar hair - new top down holographic superconducting black holes

11. Branch of black holes with no charged scalar hair - top down analogues of dilatonic black holes studied by Goldstein, Kachru, Prakash, Trivedi

Second branch appears at a higher temperature.
New consistent KK truncation of D=11 SUGRA on $\Sigma_3 \times S^4$
with $\Sigma_3 = H^3, S^3, R^3$ to an N=2 D=4 gauged SUGRA

- Rich set of solutions:

  $AdS_4 \times H^3/\Gamma \times S^4$ with N=2 susy and also N=0

  $Lif_4(39) \times H^3/\Gamma \times S^4$ with N=?

- Are these related via holographic flows?

- Have initiated a study of the N=2 SCFT at finite $T$ and $\mu$

- Shown there are new branches of black holes -- can we construct (numerically) the fully back reacted black holes? What is the phase structure? Is there holographic superconductivity? What are the ground states?

- What are underlying principles of consistent truncations?