DY PRODUCTION AT SMALL $Q_T$
AND
THE COLLINEAR ANOMALY

arXiv:1007.4005 + upcoming with Matthias Neubert

Thomas Becher
University of Bern

Seminar at Padua U., Jan. 19, 2011
OUTLINE

• Introduction
  • Drell-Yan process
  • Soft-Collinear Effective Theory
• Factorization at low transverse momentum $q_T$
  • The collinear anomaly and the definition of transverse position dependent PDFs
• Resummation of large log’s, relation to the Collins Soper Sterman (CSS) formalism
• Expansions from hell: Non-perturbative short-distance physics at low $q_T$. Numerical results
The production of a lepton pair with large invariant mass is the most basic hard-scattering process at a hadron collider.

ATLAS has 45 pb\(^{-1}\) of data: \(\sim 1.5 \times 10^7\) W’s and \(10^6\) Z’s!

This year \(\sim 1 \text{ fb}^{-1}\) should be accumulated.
ing a NNLO total power of the strong coupling constant, and di-

nel measurement. There, both the total cross section at NLO, while the second term is

where the first term is of order

result to previous measurements: the factor labeled
data well. Table I also lists four multiplicative correction

suppression of fluctuations may lead to an artificially low

care must be taken when using the data in any fits as this

which could be caused by such fluctuations. As a result,

have been suppressed; however the statistical uncertain-

fluctuations in the measured cross section in each

during the second step of the data corrections, statistical

ing introduced by the regularization condition imposed

acceptance to full 4

factor labeled

ment; the factor labeled FSR corrects for QED FSR; the

Z/

the larger mass window used in the D0 electron channel

labeled

the following text, and we provide only the central val-

the dimuon data.

To compare to the data, predictions for the

for any fit which describes the central values of the

ues without assessing possible systematic uncertainties.

the following text, and we provide only the central val-

DIRECT COMPARISONS

FIG. 2: Measurements of the normalized di-

DZero, arXiv:1006.0618, 0712.0803
DRELL-YAN PROCESSES

The production of a single electroweak boson $\gamma^*, Z, W^\pm, H$ is of great interest for

- $W$ mass and width measurements,
- PDF determinations, theory benchmarks
- Higgs discovery

Low transverse momentum $q_T$ is particularly relevant

- to extract of $W$ mass
- to reduce background for Higgs search
The perturbative expansion of the $q_T$ spectrum contains singular terms of the form ($M$ is the invariant mass of the lepton pair)

$$\frac{d\sigma}{dq_T^2} = \frac{1}{q_T^2} \left[ A^{(1)}_1 \alpha_s \ln \frac{M^2}{q_T^2} + \alpha_s A^{(1)}_0 + A^{(2)}_3 \alpha_s^2 \ln^3 \frac{M^2}{q_T^2} + \ldots \right. $$

$$+ \left. A^{(n)}_{2n-1} \alpha_s^n \ln^{2n-1} \frac{M^2}{q_T^2} + \ldots \right] + \ldots$$

which ruin the perturbative expansion at $q_T \ll M$ and must be resummed to all orders.

Classic example of an observable which needs resummation! Achieved by Collins, Soper and Sterman (CSS) ’84.
RESUMMATION

A formula which allows for the resummation of the logarithmically enhanced terms at small $q_T$ to arbitrary precision was first obtained by Collins, Soper and Sterman (CSS) in ‘84, based on earlier work of Collins and Soper.

Their is based on a factorization theorem for the cross section at low $q_T$, which Collins and Soper derived using diagrammatic techniques.

We will instead use soft-collinear effective theory to analyze the cross section.
In collider processes, we have an *interplay of three momentum regions*

- **Hard**
- **Collinear**
- **Soft**

Correspondingly, EFT for such processes has two low-energy modes:

- **Collinear fields** describing the energetic partons propagating in each direction of large energy, and
- **Soft fields** which mediate long range interactions among them.
For $M_1 \sim M_2 \ll Q$ the cross section factorizes:

$$Q \sim \frac{M_1^2}{\Lambda_s^2} \frac{M_2^2}{Q^2}$$
DIAGRAMMATIC FACTORIZATION

The simple structure of soft and collinear emissions forms the basis of the classic factorization proofs, which were obtained by analyzing Feynman diagrams.

Collins, Soper, Sterman 80’s ...

Advantages of the the SCET approach:

- Simpler to exploit **gauge invariance** on the Lagrangian level
- **Operator definitions** for the soft and collinear contributions
- Resummation with **renormalization group**
- Can include power corrections
SCET APPLICATIONS

• Charmless $B$ decays, $B \rightarrow \pi \pi$, $B \rightarrow X_u l \nu$, $B \rightarrow X_s \gamma$

• Threshold resummation / soft gluon resummation
  • DIS at large $x$, Drell-Yan rapidity spectrum, inclusive Higgs production, top production, direct photon production, single top production, $e^+e^-$ event shapes, ...

• Towards higher-log resummation $n$-jet processes
  • IR singularities of multi-loop amplitudes
  • Jet definitions, beam jets (initial state showering)
FACTORIZATION
SOFT-COLLINEAR FACTORIZATION

- Starting point is the factorization of the electroweak current in the Sudakov limit

\[ J_V^\mu \]

\[ M^2 = (p - \bar{p})^2 \]

\[ n^\mu \sim \frac{p^\mu}{p^0} \]

\[ \bar{n}^\mu \sim \frac{\bar{p}^\mu}{\bar{p}^0} \]

- In Soft-Collinear Effective Theory (SCET) this can be written in operator form as

\[ \bar{\psi} \gamma_\mu \psi \rightarrow C_V (M^2, \mu) \bar{\chi}_{hc} S^\dagger_{\bar{n}} \gamma^\mu S_n \chi_{hc} \]

SCET hc quark field
The Drell-Yan cross section is obtained from the matrix element of two currents
\[-g_{\mu\nu} \langle N_1(p) \ N_2(\bar{p}) | J_{\nu}^\mu(x) \ J_{\nu}'(0) | N_1(p) \ N_2(\bar{p}) \rangle \rightarrow \frac{1}{2N_c} |C_V(M^2, \mu)|^2\]
\[
	imes \hat{W}_{DY}(x) \langle N_1(p) | \bar{\chi}_{hc}(x) \ \frac{\not{p}}{2} \chi_{hc}(0) | N_1(p) \rangle \langle N_2(\bar{p}) | \bar{\chi}_{hc}(0) \ \frac{\not{p}}{2} \chi_{hc}(x) | N_2(\bar{p}) \rangle
\]
\(n\) and \(\bar{n}\) are light-cone reference vectors along \(p\) and \(\bar{p}\).

The soft function contains a product of four Wilson lines along the directions of large energy flow
\[
\hat{W}_{DY}(x) = \frac{1}{N_c} \langle 0 | \text{Tr} \left[ S_n^\dagger(x) \ S_{\bar{n}}(x) \ S_{\bar{n}}^\dagger(0) \ S_n(0) \right] | 0 \rangle
\]
\[
S_n(x) = P \exp \left[ i \int_{-\infty}^{0} ds \ n \cdot A_s(x + sn) \right]
\]
DERIVATIVE EXPANSION

Final step is to expand the matrix elements in small momentum components, i.e. to perform a derivative expansion.

The light-cone components \((n \cdot k, \bar{n} \cdot k, k_\perp)\) scale as

\[
p_{hc} \sim M (\lambda^2, 1, \lambda), \quad p_{\bar{hc}} \sim M (1, \lambda^2, \lambda).
\]

\[
p_s \sim M (\lambda^2, \lambda^2, \lambda^2).
\]

while the separation between the two currents scales as

\[
x \sim M^{-1}(1, 1, \lambda^{-1})
\]

\[
A^\mu_s(x) = A^\mu_s(0) + x \cdot \partial A^\mu_s(0) + \ldots
\]

power suppressed, can be dropped
NAIVE FACTORIZATION

Dropping power suppressed $x$-dependence leads to the result

$$\hat{W}_{DY}(0) \langle N_1(p) | \bar{\chi}_{hc}(x_+ + x_\perp) \frac{\hat{p}}{2} \chi_{hc}(0) | N_1(p) \rangle \langle N_2(\bar{p}) | \bar{\chi}_{hc}(0) \frac{\hat{p}}{2} \chi_{hc}(x_+ + x_\perp) | N_2(\bar{p}) \rangle$$

$$\nearrow \times \text{“transverse PDF”} \times \text{“transverse PDF”}$$

KLN cancellation!

this spells trouble: well known that transverse PDF not well defined w/o additional regulators

For comparison: for soft-gluon resummation, the result is

$$\hat{W}_{DY}(x_0) \langle N_1(p) | \bar{\chi}_{hc}(x_+) \frac{\hat{p}}{2} \chi_{hc}(0) | N_1(p) \rangle \langle N_2(\bar{p}) | \bar{\chi}_{hc}(0) \frac{\hat{p}}{2} \chi_{hc}(x_-) | N_2(\bar{p}) \rangle$$

“soft” $\times$ “standard PDF” $\times$ “standard PDF”
CROSS SECTION

In terms of the hadronic matrix elements

\[ \mathcal{B}_{q/N}(z, x_T^2, \mu) = \frac{1}{2\pi} \int dt \, e^{-izt\vec{n} \cdot \vec{p}} \langle N(p) | \bar{\chi}(t\vec{n} + x_\perp) \frac{\vec{\eta}}{2} \chi(0) | N(p) \rangle \]

on then obtains the DY cross section

\[ \frac{d^3\sigma}{dM^2 \, dq_T^2 \, dy} = \frac{4\pi \alpha^2}{3N_c M^2 s} \left| C_V(-M^2, \mu) \right|^2 \frac{1}{4\pi} \int d^2x_\perp \, e^{-iq \cdot x_\perp} \times \sum_q e_q^2 \left[ \mathcal{B}_{q\rightarrow N_1}(\xi_1, x_T^2, \mu) \mathcal{B}_{\bar{q}\rightarrow N_2}(\xi_2, x_T^2, \mu) + (q \leftrightarrow \bar{q}) \right] + \mathcal{O} \left( \frac{q_T^2}{M^2} \right) \]

with

\[ \xi_1 = \sqrt{\tau} e^y, \quad \xi_2 = \sqrt{\tau} e^{-y}, \quad \text{with} \quad \tau = \frac{m_\perp^2}{s} = \frac{M^2 + q_T^2}{s}. \]
CROSS SECTION

\[
\frac{d^3 \sigma}{dM^2 \, dq_T^2 \, dy} = \frac{4\pi \alpha^2}{3N_c M^2 s} \left| C_V(-M^2, \mu) \right|^2 \frac{1}{4\pi} \int d^2 x_\perp \, e^{-iq_\perp \cdot x_\perp} \times \sum_q e_q^2 \left[ B_{q/N_1}(\xi_1, x_T^2, \mu) \, B_{\bar{q}/N_2}(\xi_2, x_T^2, \mu) + (q \leftrightarrow \bar{q}) \right] + O\left( \frac{q_T^2}{M^2} \right)
\]

The resummation would then be obtained by solving the RG equation

\[
\frac{d}{d \ln \mu} \, C_V(M^2, \mu) = \left[ \Gamma_{\text{cusp}}^{F}(\alpha_s) \ln \frac{-M^2}{\mu^2} + 2\gamma_q(\alpha_s) \right] \left. C_V(M^2, \mu) \right|
\]

see SCET papers: Gao, Li, Liu 2005; Idilbi, Ji, Yuan 2005; Mantry, Petriello 2009

This cannot be correct! If \( B_{q/N_1}(\xi_1, x_T^2, \mu) \, B_{\bar{q}/N_2}(\xi_2, x_T^2, \mu) \) is independent of \( M \), the above cross section is \( \mu \) dependent!
COLLINEAR ANOMALY

RG invariance of cross section implies that the product of transverse PDFs $B_{q/N_1}(\xi_1, x_T^2, \mu) B_{\bar{q}/N_2}(\xi_2, x_T^2, \mu)$ must contain hidden $M^2$ dependence.

Analyzing the relevant diagrams, one finds that an additional regulator is needed to make transverse PDFs well defined. In the product of the two PDFs, this regulator can be removed, but anomalous $M^2$ dependence remains. Can refactorize

$$[B_{q/N_1}(z_1, x_T^2, \mu) B_{\bar{q}/N_2}(z_2, x_T^2, \mu)]_{M^2} = \left(\frac{x_T^2 M^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T^2, \mu)} B_{q/N_1}(z_1, x_T^2, \mu) B_{\bar{q}/N_2}(z_2, x_T^2, \mu),$$

with $\frac{dF_{q\bar{q}}(x_T^2, \mu)}{d\ln \mu} = 2\Gamma^F_{\text{cusp}}(\alpha_s)$

Note that $M^2$ dependence exponentiates!
Regular soft-collinear factorization:

\[ J(M_1^2, \mu) \rightarrow H(Q^2, \mu) \rightarrow J(M_2^2, \mu) \]

\[ S(\Lambda_s^2, \mu) \]

Anomalous factorization

\[ B_{q/N_1}(x_\perp, \mu) \rightarrow H(Q^2, \mu) \rightarrow B_{\bar{q}/N_2}(x_\perp, \mu) \]

\[ F(x_\perp, \mu) \ln(x_\perp Q^2) \]
TRANSVERSE PDFs

What God has joined together, let no man separate...

The “operator definition of TMD PDFs is quite problematic [...] and is nowadays under active investigation”.

quote from Cherednikov and Stefanis ’09. For reviews, see Collins ’03, ’08

Regularization of the individual transverse PDFs is delicate, but the product is well defined, and has specific dependence on the hard momentum transfer $M^2$.

Anomaly: Classically, $\langle N_1(p) | \bar{\chi}_{hc}(x_+ + x_\perp) \bar{\pi} \chi_{hc}(0) | N_1(p) \rangle$ is invariant under a rescaling of the momentum of the other nucleon $N_2$. Quantum theory needs regularization. Symmetry cannot be recovered after removing regulator.

Not an anomaly of QCD, but of the low energy theory.
RESUMMATION
SIMPLIFICATION FOR $q_T^2 \gg \Lambda_{\text{QCD}}$

For perturbative values of $q_T$ we can perform an operator product expansion

$$\mathcal{B}_{i/N}(\xi, x_T^2, \mu) = \sum_j \int_{\xi}^{1} \frac{dz}{z} \mathcal{I}_{i\leftarrow j}(z, x_T^2, \mu) \phi_{j/N}(\xi/z, \mu) + \mathcal{O}(\Lambda_{\text{QCD}}^2 x_T^2)$$

Again, only the product of two $\mathcal{I}_{i\leftarrow j}(z, x_T^2, \mu)$ functions is well defined.

$$[\mathcal{I}_{q\leftarrow i}(z_1, x_T^2, \mu) \mathcal{I}_{\bar{q}\leftarrow j}(z_2, x_T^2, \mu)]_{q^2} = \left(\frac{x_T^2 q^2}{4e^{-2\gamma_E}}\right)^{-F_{qq}(x_T^2, \mu)} I_{q\leftarrow i}(z_1, x_T^2, \mu) I_{\bar{q}\leftarrow j}(z_2, x_T^2, \mu)$$

Effective theory diagrams for $\mathcal{I}_{i\leftarrow j}(z, x_T^2, \mu)$ are not well-defined in dim. reg.. Following Smirnov '83, we use additional analytical regularization, which is very economical, since it does not introduce additional scales into the problem.
ANALYTICAL REGULARIZATION

- Raise QCD propagators carrying large momentum $p$, $(\vec{p})$ to fractional powers $\alpha$, $(\beta)$:

$$\frac{1}{-(p-k)^2 - i\varepsilon} \rightarrow \frac{\nu_1^{2\alpha}}{[-(p-k)^2 - i\varepsilon]^{1+\alpha}}$$

- Limit $\alpha \rightarrow 0, \beta \rightarrow 0$ is trivial for QCD, but effective theory diagrams have poles, which only cancel in the sum of collinear and anti-collinear diagrams.
ANALYTICAL REGULARIZATION

- Regulators play double role. E.g. $\alpha$ regulates $hc$ propagators and $\overline{hc}$ Wilson line

\[
\frac{n^\mu}{n \cdot k - i\varepsilon} \rightarrow \nu_1^{2\alpha} \frac{n^\mu \bar{n} \cdot p}{(n \cdot k \bar{n} \cdot p - i\varepsilon)^{1+\alpha}}
\]

- Regulator breaks invariance of anti-hard-collinear sector under a rescaling of the hard-collinear momentum $p \rightarrow \lambda p$. 

25
I-LOOP RESULT

Taking first $\beta \to 0$, then $\alpha \to 0$, one finds ($L_\perp = \ln \frac{x_T^2 \mu^2}{4e^{-2\gamma E}}$)

$$\mathcal{I}_{q\to q}(z, x_T^2, \mu) = \delta(1 - z) - \frac{C_F \alpha_s}{2\pi} \left\{ \left( \frac{1}{\epsilon} + L_\perp \right) \left[ \left( \frac{2}{\alpha} - 2 \ln \frac{\mu^2}{\nu_1^2} \right) \delta(1 - z) + \frac{1 + z^2}{(1 - z)_+} \right] \\
+ \delta(1 - z) \left( -\frac{2}{\epsilon^2} + L_\perp^2 + \frac{\pi^2}{6} \right) - (1 - z) \right\}$$

anomalous $M^2$ dep.

$$\mathcal{I}_{\bar{q}\to q}(z, x_T^2, \mu) = \delta(1 - z) - \frac{C_F \alpha_s}{2\pi} \left\{ \left( \frac{1}{\epsilon} + L_\perp \right) \left[ \left( -\frac{2}{\alpha} + 2 \ln \frac{M^2}{\nu_1^2} \right) \delta(1 - z) \\
+ \frac{1 + z^2}{(1 - z)_+} \right] - (1 - z) \right\}$$

In the product the $1/\alpha$ divergences vanish, but anomalous $M^2$ dependence remains.
• For the functions $I$ and $F$, one then obtains

$$F_{qq}(L_\perp, \alpha_s) = \frac{C_F \alpha_s}{\pi} L_\perp + \mathcal{O}(\alpha_s^2)$$

$$I_{q\rightarrow q}(z, L_\perp, \alpha_s) = I_{\bar{q}\rightarrow q}(z, L_\perp, \alpha_s) = \delta(1 - z) \left[ 1 + \frac{C_F \alpha_s}{4\pi} \left( L_\perp^2 + 3L_\perp - \frac{\pi^2}{6} \right) \right]$$

$$- \frac{C_F \alpha_s}{2\pi} \left[ L_\perp P_{q\rightarrow q}(z) - (1 - z) \right] + \mathcal{O}(\alpha_s^2)$$

$$I_{q\rightarrow g}(z, L_\perp, \alpha_s) = I_{\bar{q}\rightarrow g}(z, L_\perp, \alpha_s) = -\frac{T_F \alpha_s}{2\pi} \left[ L_\perp P_{q\rightarrow g}(z) - 2z(1 - z) \right] + \mathcal{O}(\alpha_s^2)$$

• Solving its RG and using Davies, Stirling and Webber ’84 and de Florian and Grazzini ’01, we extract the two-loop $F$

$$F_{qq}(L_\perp, \alpha_s) = \frac{\alpha_s}{4\pi} \Gamma_0^F L_\perp + \left( \frac{\alpha_s}{4\pi} \right)^2 \left[ \frac{\Gamma_0^F \beta_0}{2} L_\perp^2 + \Gamma_1^F L_\perp + d_2^q \right]$$

$$d_2^q = C_F \left[ C_A \left( \frac{808}{27} - 28\zeta_3 \right) - \frac{224}{27} T_F n_f \right]$$

Casimir scaling

$$\frac{F_{qq}(L_\perp, \alpha_s)}{C_F} = \frac{F_{gg}(L_\perp, \alpha_s)}{C_A}$$

All the necessary input for NNLL resummation!
RESUMMED RESULT

\[
\frac{d^3 \sigma}{dM^2 dq_T^2 dy} = \frac{4\pi \alpha^2}{3N_c M^2 s} \sum_q e_q^2 \sum_{i=q,g,j=\bar{q},g} \sum \int_{\xi_1}^1 \frac{dz_1}{z_1} \int_{\xi_2}^1 \frac{dz_2}{z_2} \partial_{q,i} \left( \frac{\xi_1}{z_1}, \frac{\xi_2}{z_2}, q^2_T, M^2, \mu \right) \phi_{i/N_1}(z_1, \mu) \phi_{j/N_2}(z_2, \mu) + (q, i \leftrightarrow \bar{q}, j) \right]
\]

The hard-scattering kernel is

\[
C_{q\bar{q} \rightarrow ij}(z_1, z_2, q^2_T, M^2, \mu) = |C_V(-M^2, \mu)|^2 \frac{1}{4\pi} \int d^2 x_\perp e^{-iq_\perp \cdot x_\perp} \left( \frac{x_T^2 M^2}{4e^{-2\gamma E}} \right)^{-F_{q\bar{q}}(x_T^2, \mu)} \times I_{q-i}(z_1, x_T^2, \mu) I_{\bar{q}-j}(z_2, x_T^2, \mu)
\]

- Two sources of \( M \) dependence: hard function and anomaly
- Fourier transform can be evaluated numerically or analytically, if higher-log terms are expanded out.
The low scale is $\mu_b = b_0/x_T$, and we set $b_0 = 2e^{-\gamma E}$.

Landau-pole singularity in the Fourier transform. To use the formula, one needs additional prescription to deal with this.
RELATION TO CSS

If adopt the choice $\mu = \mu_b = 2e^{-\gamma_E} x_\perp$ in our result reduces to CSS formula, provided we identity

\[ A(\alpha_s) = \Gamma_{\text{cusp}}^F(\alpha_s) - \frac{\beta(\alpha_s)}{2} \frac{dg_1(\alpha_s)}{d\alpha_s}, \]

\[ B(\alpha_s) = 2\gamma^q(\alpha_s) + g_1(\alpha_s) - \frac{\beta(\alpha_s)}{2} \frac{dg_2(\alpha_s)}{d\alpha_s}, \]

\[ C_{ij}(z, \alpha_s(\mu_b)) = |C_V(-\mu_b^2, \mu_b)| I_{i\rightarrow j}(z, 0, \alpha_s(\mu_b)), \]

Use these relations to derive unknown three-loop coefficient, necessary for NNLL resummation

\[ A^{(3)} = \Gamma_2^F + 2\beta_0 d_2^q = 239.2 - 652.9 \]

Not equal to the cusp anom. dim. as was usually assumed!
DIVERGENT EXPANSIONS, AND OTHER SURPRISES
The spectrum has a number of quite remarkable features which we now discuss in turn:

- Expansion in $\alpha_s$: strong factorial divergence
- $q_T$-spectrum:
  - calculable, even near $q_T = 0$
  - expansion around $q_T = 0$: worst expansion, ever
- Long-distance effects associated with $\Lambda_{QCD}$
  - small, but OPE breaks down
LEADING MOMENTUM DEPENDENCE

Up to corrections suppressed by powers of $\alpha_s$, the $q_T$-dependence of our formula result has the form

$$\frac{1}{4\pi} \int d^2x_\perp e^{-iq_\perp \cdot x_\perp} e^{-\eta L_\perp - \frac{1}{4}aL^2_\perp} \equiv \frac{e^{-2\gamma_E}}{\mu^2} K(\eta, a, \frac{q_T^2}{\mu^2})$$

with $L_\perp = \ln \frac{x_T^2\mu^2}{4e^{-2\gamma_E}}$, and the two quantities

$$\eta = \frac{C_F\alpha_s}{\pi} \ln \frac{M^2}{\mu^2} = \mathcal{O}(1) \quad \text{and} \quad a = \alpha_s(\mu) \times \mathcal{O}(1).$$
PERTURBATIVE EXPANSION

Since \( L_\perp \) is not large as long as \( \mu \approx q_T \), the \( aL^2_\perp \) term can formally be expanded away

\[
K(\eta, a, r) = \left[ 1 - \frac{a}{4} \partial^2_\eta + \frac{1}{2!} \left(\frac{a}{4}\right)^2 \partial^4_\eta + \ldots \right] K(\eta, 0, r)
\]

and for \( a=0 \), we can perform the integral explicitly:

\[
K(\eta, 0, r) = r^{\eta-1} \frac{\Gamma(1 - \eta)}{e^{2(\eta-1)\gamma_E} \Gamma(\eta)}
\]
FACTORIAL DIVERGENCE

Unfortunately, this series is strongly factorially divergent:

\[
K(\eta, a, 1)_{\text{exp}} = \sum_{n=0}^{\infty} \frac{(2n)!}{n!} \left( -\frac{a}{4} \right)^n \left[ \frac{1}{(1 - \eta)^{2n+1}} - e^{-2\gamma E} \right] + \ldots
\]

first noted by: Frixione, Nason, Ridolfi ‘99

Can Borel resum it, which leads makes nonperturbative and highly nontrivial \(a\) dependence explicit

\[
K(\eta, a, 1)_{\text{Borel}} = \sqrt{\frac{\pi}{a}} \left\{ e^{\frac{(1-\eta)^2}{a}} \left[ 1 - \text{Erf} \left( \frac{1 - \eta}{\sqrt{a}} \right) \right] - e^{-2\gamma E + \frac{1}{a}} \left[ 1 - \text{Erf} \left( \frac{1}{\sqrt{a}} \right) \right] \right\} + \ldots
\]

In practice, it is simplest, to use the exact expression and evaluate \(K\)-function numerically.
VERY LOW $q_T$

For moderate $q_T$, the natural scale choice is $\mu = q_T$. However, detailed analysis shows that near $q_T \approx 0$ the Fourier integral is dominated by

$$\langle x_T^{-1} \rangle = q_* = M \exp \left( -\frac{\pi}{2C_F\alpha_s(q_*)} \right) = 1.75 \text{ GeV \ for } M = M_Z$$

which corresponds to $\eta = 1$.

→ Spectrum can be computed with short-distance methods down to $q_T=0$!
INTERCEPT AT $Q_T=0$

- Dedicated analysis of $q_T \rightarrow 0$ limit yields:

$$\frac{d\sigma}{dq_T^2} \sim \frac{\mathcal{N}}{\sqrt{\alpha_s}} e^{-\#/\alpha_s} \left( 1 + c_1 \alpha_s + \ldots \right)$$

- Were for the first time we are able to compute the normalization $\mathcal{N}$ and NLO coefficient $c_1$

- Expression cannot be expanded about $\alpha_s = 0$ (essential singularity)

Parisi, Petronzio 1979; Collins, Soper, Sterman 1985; Ellis, Veseli 1998
SLOPE AT $Q_T=0$?

Given our result for the intercept, we can also try to obtain derivatives with respect to $q_T^2$. Leading term is obtained by expanding

$$\frac{1}{4\pi} \int d^2x_\perp e^{-iq_\perp \cdot x_\perp} e^{-\eta L_\perp} - \frac{1}{4} a L_\perp^2 \equiv \frac{e^{-2\gamma_E}}{\mu^2} K(\eta, a, \frac{q_T^2}{\mu^2})$$

Yields extremely violently divergent series

$$K(\eta = 1, a, q_T)\big|_{\text{exp}} \sim \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{a}} e^{n^2/a} \left(\frac{q_T^2}{q_*^2}\right)^{n-1}$$
NUMERICAL RESULTS

• Spectrum can be calculated numerically, even though power expansion in $q_T^2$ is absolutely meaningless (not even Borel summable)!

• Find smooth behavior down to very small $q_T$
• Spectrum is short-distance dominated, but essential features are non-perturbative!
LOW-ENERGY QCD EFFECTS

• Related question is that about the impact of long-distance power correction in matching relation

\[ B_{i/N}(\xi, x_T^2, \mu) = \sum_j \int_\xi^1 dz I_{i-j}(z, x_T^2, \mu) \phi_{j/N}(\xi/z, \mu) + O(\Lambda_{\text{QCD}}^2 x_T^2) \]

• Find that these cannot be analyzed order by order, but only numerically using functions that vanish at large \(x_T^2\), such as \(\theta(1 - \Lambda^2 x_T^2)\) or \(e^{-\Lambda^2 x_T^2}\)

• Fixed-order OPE in \(x_T^2\) is again extremely divergent. Same as expansion of \(K\) around \(q_T=0\)
LONG-DISTANCE EFFECTS

- Yet resummed behavior is smooth and rather insensitive to the way in which the cutoff is introduced:

- Indications that long-distance effects are very small already above $q_T=2$ GeV
Resulting power correction has a complicated shape, but is approximately linear:

\[ \sim \left( 1 - 0.7 \times \frac{\Lambda_{NP}}{q_T} \right) \]

Cannot be described in terms of a single operator matrix element
CONCLUSION

• Have derived resummed result for Drell-Yan $q_T$ spectrum
  • Naive factorization broken by collinear anomaly. Only the product of two transverse PDFs is well defined, but has anomalous dependence on the large momentum transfer
  • Reduces to the known CSS result for a special scale choice.
  • Obtain three-loop coefficient $A^{(3)}$, the last missing piece needed for NNLL accuracy.
• Many surprising features:
  • emergence of nonperturbative scale $q^* \sim 2\text{GeV}$: spectrum is short-distance dominated, even at very low $q_T$
  • strongly divergent expansions in $\alpha_s$, $q_T/q^*$, $\Lambda_{\text{QCD}}/q^*$.
• Phenomenological analysis at NNLL+NLO is in progress.