BACKGROUND EXPERIENCE AND THEORY INNOVATION FOR LHC

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A theory revolution

- Deeper understanding of the structure of gauge theories
- Sharp theoretical predictions for collider experiments
- A new technical revolution and a pace of progress to be very proud of
Revolutionary new methods for one-loop calculations and a promise for precise multi-particle production cross-sections at the Tevatron and the LHC

Impressive progress on NNLO methods which has lead to precision phenomenology for LEP, HERA, Tevatron and the LHC
One-loop amplitudes

Jet physics at LEP, strong coupling

NNLO theory

DIS at HERA

JET ALGORITHMS

Final states with many particles at the LHC

PDFs for the Tevatron and the LHC

Drell-Yan and Higgs @ Tevatron/LHC

The string connection
One-loop amplitudes from trees... and masters!!!
One-loop amplitude in Gauge theory

Integrals in scalar field theory

Known method(s) to compute $a, b, c, d$ coefficients had a ($\#$ Legs)! computational cost
Unitarity

Bern, Dixon, Dunbar, Kosower 1990s

\[ \approx \int \frac{d^d k}{k^2 (k + p)^2} \]

- Trees as input for the integrand
- Manifest gauge invariance cancelations
- Simplifications by using “natural” spinor variables

- Mismatch between Trees in four dimensions and loop integration in D-dimensions
- Introduction of four dimensional helicity regularization scheme
- Clever theory input (collinear factorization) to recover the full one-loop amplitude

Trees were an essential ingredient. No explicit connection of master integral coefficients to tree amplitudes.
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Coefficient of box master!

\[ C_4 = C_4 + \ldots \]

- Simple product of four tree amplitudes
- Evaluated at complex momenta
- Corresponding to loop momentum values where all propagators of the box master integral are ON-SHELL

Britto, Cachazo, Feng 2004
\[
\int \frac{d^d k}{(2\pi)^d} \left[ c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) \\
+ \tilde{c}_4 \tilde{f}_4(\vec{k}) + \tilde{c}_3 \tilde{f}_3(\vec{k}) + \tilde{c}_2 \tilde{f}_2(\vec{k}) + \tilde{c}_1 \tilde{f}_1(\vec{k}) \right]
\]
\[ \int \frac{d^d k}{(2\pi)^d} \left[ c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) ight] \]

After Integration:
ONE-LOOP INTEGRAND

Ossola, Papadopoulos, Pittau 2006
(building on del Aguila, Pittau, 2004)

\[
\int \frac{d^d k}{(2\pi)^d} \left[ c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) \right]
\]

After Integration:

\[
c_4
\]
Ossola, Papadopoulos, Pittau 2006
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\[
\int \frac{d^d k}{(2\pi)^d} \left[ c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) \right]
\]

After Integration:

\[
c_4 + c_3
\]
\[ \int \frac{d^d k}{(2\pi)^d} \left[ c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) \right] \]

After Integration:

\[ = c_4 + c_3 + c_2 \]
Ossola, Papadopoulos, Pittau 2006
(building on del Aguila, Pittau, 2004)

\[ \int \frac{d^d k}{(2\pi)^d} \left[ c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) \right] \]

After Integration:

\[ = c_4 + c_3 + c_2 + c_1 \]
\[ \int \frac{d^d k}{(2\pi)^d} \left[ c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) ight. \\
\left. + \tilde{c}_4 \tilde{f}_4(\vec{k}) + \tilde{c}_3 \tilde{f}_3(\vec{k}) + \tilde{c}_2 \tilde{f}_2(\vec{k}) + \tilde{c}_1 \tilde{f}_1(\vec{k}) \right] \]
\[ \int \frac{d^d k}{(2\pi)^d} \left[ c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) ight. 
+ \tilde{c}_4 \tilde{f}_4(\vec{k}) + \tilde{c}_3 \tilde{f}_3(\vec{k}) + \tilde{c}_2 \tilde{f}_2(\vec{k}) + \tilde{c}_1 \tilde{f}_1(\vec{k}) \left] \right. 
\]

\[ \tilde{f}_i(\vec{k}), f_i(\vec{k}) : \text{Known rational functions of the loop momentum} \]
\[
\int \frac{d^d k}{(2\pi)^d} \left[ c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) 
+ \tilde{c}_4 \tilde{f}_4(\vec{k}) + \tilde{c}_3 \tilde{f}_3(\vec{k}) + \tilde{c}_2 \tilde{f}_2(\vec{k}) + \tilde{c}_1 \tilde{f}_1(\vec{k}) \right]
\]

\(\tilde{f}_i(\vec{k}), f_i(\vec{k})\): Known rational functions of the loop momentum

\(\tilde{c}_i, c_i\): coefficients can be determined algebraically computing the integrand at a sufficient number of values for \(\vec{k}\)
\[
\int \frac{d^d k}{(2\pi)^d} \left[ c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) \right. \\
\left. + \tilde{c}_4 \tilde{f}_4(\vec{k}) + \tilde{c}_3 \tilde{f}_3(\vec{k}) + \tilde{c}_2 \tilde{f}_2(\vec{k}) + \tilde{c}_1 \tilde{f}_1(\vec{k}) \right]
\]
Integrand is “easy”, essentially a tree amplitude
\[
\int \frac{d^d k}{(2\pi)^d} \left[ c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) 
+ \tilde{c}_4 \tilde{f}_4(\vec{k}) + \tilde{c}_3 \tilde{f}_3(\vec{k}) + \tilde{c}_2 \tilde{f}_2(\vec{k}) + \tilde{c}_1 \tilde{f}_1(\vec{k}) \right] = \int \frac{d^d k}{(2\pi)^d}
\]

Integrand is “easy”, essentially a tree amplitude

Evaluate integrand at loop momenta values such as loop particles are set ON SHELL
Integrand is “easy”, essentially a tree amplitude

Evaluate integrand at loop momenta values such as loop particles are set ON SHELL

ON-SHELL: determines coefficients successively
Coefficients as tree products

ON-SHELL loop propagators = Product of tree amplitudes

Evaluation of trees with powerful recursive methods

e.g. Berends-Giele, Britto-Cachazo-Feng-Witten, etc
Conflict of dimensions

Loop Integrations in D dimensions, Tree amplitudes in four dimensions. Mismatch, i.e. missing terms from amplitude evaluation. Requires a second calculation.

- Specialized tree-like recursions in $D=4$ for the missing terms
  Berger, Bern, Dixon, Forde, Kosower 2006

- Elegant/general solution: Amplitude in a general dimension from results in $D=5$ and $D=6$.
  Ellis, Giele, Kunszt, Melnikov 2008

- Specialized Feynman rules for missing terms:
  Draggiotis, Garzelli, Papadopoulos, Pittau 2009
Breathtaking developments

One-loop amplitudes with 22 gluons Giele, Zanderighi (08); Lazopoulos (08); Giele, Winter (09)

numerical evaluation of all 2 to 4 amplitudes in the Les-Houches 2007 wish-list van Hameren, Papadopoulos, Pittau (09)

\[ q\bar{q}, gg \rightarrow tt\bar{b}b, b\bar{b}b\bar{b}, W^+W^−b\bar{b}, ttgg \]

\[ q\bar{q}' \rightarrow Wggg, Zggg \]
W+3 jets: NLO cross-section

Large $N_c$ approximation
Ellis, Giele, Kunszt, Melnikov, Zanderighi;
Berger, Bern, Dixon, Cordero, Forde,
Gleisberg, Ita, Kosower, Maitre

NEW: complete NLO

Start of a new era, with precise theoretical predictions for multiparticle production at the LHC
$pp \rightarrow t\bar{t}b\bar{b}$: NLO cross-section

Brendenstein, Denner, Dittmaier, Pozzorini
First full NLO calculation for a 2 to 4 process at a hadron collider

Important Higgs boson background

With Feynman diagrams

Intelligent, mostly numerical reduction, to master integrals

Exploits infrared regulators other than the dimension

And new methods
Bevilacqua, Czakon, Papadopoulos, Pittau, Worek

very large NLO corrections
What can we hope for?

We cannot do better than tree calculations..., i.e. processes with 7 or 8 particles in the final state.

All 2 to 4 processes with both Feynman diagrammatic and unitarity methods

2 to 5 and perhaps 2 to 6 processes with unitarity methods
Jets and Infrared safety

- $\infty$ + $\infty$ = F\(\text{IN\text{ITE}}\)

Soft or Collinear parton emission must not alter the number of jets in an event.
Many jet measurements are not directly comparable to perturbative calculations (e.g. W+3 jets with JETCLU @ NLO)
To profit from NLO advances: infrared safe algorithms
Fast and Safe Jet Finding

Cacciari, Salam, Soyez (2007-2009)

Fast implementation of recombination algorithms

New infrared safe cone algorithm (SISCone)

Better understanding of jet areas

anti-Kt: recombination algorithm with “perfect cones”
SubJets and Higgs Searches

Butterworth, Davison, Rubin, Salam (2008)

Heavy Jet from the decay of a high pt Higgs boson has a characteristic substructure

Jet algorithms have varied diagnostic power

DISCOVERY CHANNEL AT THE LHC

Similar approach for ttH production

Dienstag, 19. Januar 2010
The NNLO front

Precision of measurements at collider experiments is often excellent.

Perturbation theory is often slow at work, first correction after the leading order too large and too uncertain.

All “2 to 1” and “2 to 2” hadron collider processes must be computed at NNLO.

LEP, HERA, TEVATRON, LHC data = NNLO phenomenology
Three-jet events from LEP

LEP Legacy: Excellent measurements of three jet cross-sections and jet event shapes at various energies.

Precise extraction of the strong coupling constant; largest error from theoretical prediction of the cross-section.

NNLO corrections to $e^+e^- \rightarrow 3\text{jets}$ was the holy grail of the QCD community for more than a decade.
Cancelation of singularities

Two-loop amplitude computed already in 2001 by Garland, Gehrmann, Glover, Koukoutsakis, Remiddi

A universal method for the cancelation of matrix element singularities through NNLO for lepton collider processes by Gehrmann-de Ridder, Gehrmann, Glover, Heinrich (2007)

Revision and an intricate correction by Weinzierl (2008).
\( \alpha_s \) from jet event shapes

A synthesis of fixed order QCD, Electroweak corrections, resummation, and hadronization effects describe excellently three jet events at LEP.

State of the art extraction of alphas with the NNLO result + NLL resummation

\[ \alpha_s(M_Z) = 0.1224 \pm 0.0009 \text{(stat)} \pm 0.0009 \text{(exp)} \pm 0.0012 \text{(had)} \pm 0.0035 \text{(theo)} \]

also from NNLO+"SCET resummation" of the thrust distribution (Becher, Schwarz).
Legacy of HERA

Tremendous contributions in understanding QCD and the proton Altarelli-Parisi evolution kernels computed through NNLO, and structure functions through NNNLO! Moch, Vogt, Vermaseren [2004, 2006, 2009]

Experimental highlight: measurement of $F_L$, directly sensitive to the gluon density.

\[ F_L(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} x^2 \int_x^1 \frac{dy}{y^3} \left[ \frac{8}{3} F_2(y, Q^2) + \frac{40}{9} y g(y, Q^2)(1 - \frac{x}{y}) \right] \]
Several efforts (CTEQ, MSTW, Alekhin, HERA collaborations) have updated parton densities: input for precise hadron collider phenomenology.

New ideas on pdf extraction, using Artificial Neural Network methods Ball, Del Debbio, Forte, Guffanti, Latorre, Piccione, Rojo, Ubiali.

Improvements on theoretical treatment, better error estimation, but also important changes from older sets.
Parton Densities

pdf uncertainties have surprised us at times

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comparable or bigger uncertainty than scale choice

new: estimate of $\alpha_s$ uncertainty (Martin, Stirling, Thorne, Watt)

$389.0 \text{ fb} +8.1\% \text{ (scale)} +13.6\% \text{ (90\% CL)}$ (Mhiggs = 165 GeV)

@ TEVATRON
Drell-Yan

Clean signal and high precision measurements at a hadron collider environment

Luminosity Monitor

Parton densities

W-mass, Weinberg angle

NEW W-MASS MEASUREMENT FROM D0
Drell-Yan theory

**NNLO total cross-section**
Hamberg, van Neerven 1990; Harlander, Kilgore 2002

**NNLO rapidity distribution**
CA, Dixon, Menikov, Petriello 2004

**Fully differential NNLO**
Melnikov, Petriello 2006; Catani, Cieri, Ferrera, Grazzini 2009

\[ \text{NEXT(?) : W-mass measurement requires mixed QCDxQED corrections} \]
Higgs via gluon fusion

Total [Harlander, Kilgore 02; CA, Melnikov 02; Ravindran, Smith 03] and fully differential cross-sections through NNLO [CA, Melnikov, Petriello 04; CA, Dissertori, Stockli 07; Catani, Grazzini 07]

Very large perturbative corrections, which are sensitive to selection cuts
CDF and D0 start placing limits on higgs boson cross-sections

Exclusion with a detailed comparison of data with signal and background distributions

Requires incredible control over qcd effects

\[ \sigma_{\text{NLO}} = 81\% \sigma_{\text{NNLO}} \]
\[ \sigma_{\text{LO}} = 38\% \sigma_{\text{NNLO}} \]
Higgs signal selection

- Break up total nnlo cross-section into 0, 1, and 2 jet bins (Pt, jet = 20 gev). Theory precision degrades from the 0-jet to the 1-jet and the 2-jet sample.

\[
\frac{\Delta N_{\text{inc}}(\text{scale})}{N_{\text{inc}}} = 66.5\% \cdot \left(\frac{+5\%}{-9\%}\right) + 28.6\% \cdot \left(\frac{+24\%}{-22\%}\right) + 4.9\% \cdot \left(\frac{+78\%}{-41\%}\right) = \left(\frac{+14.0\%}{-14.3\%}\right)
\]

- Apply slightly different e.g. lepton selections in the various jet-bins, which are more severe in the 0-jet bin.

\[
\frac{\Delta N_{\text{signal}}(\text{scale})}{N_{\text{signal}}} = 60\% \cdot \left(\frac{+5\%}{-9\%}\right) + 29\% \cdot \left(\frac{+24\%}{-22\%}\right) + 11\% \cdot \left(\frac{+78\%}{-41\%}\right) = \left(\frac{+18.5\%}{-16.3\%}\right)
\]

- Theory uncertainty for the accepted signal events is different than for the total number before cuts.

(CA, Dissertori, Grazzini, Stoeckli, Webber)

Dienstag, 19. Januar 2010
Lesson: check theory uncertainty on the kinematic bins which drive exclusion of an NNLO computation of a neural net is as simple as for a rapidity distribution. (CA, Dissertori, Grazzini, Stoeckli, Webber)

Highly recommended for the CDF and D0 analyses.
Lesson: check theory uncertainty on the kinematic bins which drive exclusion of an NNLO computation of a neural net is as simple as for a rapidity distribution. (CA,Dissertori,Grazzini,Stoeckli, Webber)

Highly recommended for the CDF and DO analyses.
Latest exclusion limits

Mature analysis, with many improvements concerning the treatment of theory uncertainties

Space for further improvements

Using little theory input is a virtue for an experimental study.

Little theory input should not mean idealized theory input (total cross-section)
Iterative perturbation series

The perturbation series of gauge theories displays cross-order iterations.

These are needed to cancel infrared and UV divergences, filtering the superposition principle from ultra short and very large distance effects.

They are exploited to formulate parton shower algorithms, and resumming large logarithms.

But, the remainder seems very different at each order in perturbation theory!
An unexpected iteration in N=4 super Yang-Mills theory

\[ \mathcal{M}_4^{(2)}(\epsilon) = \frac{1}{2} \left( \mathcal{M}_4^{(1)}(\epsilon) \right)^2 + f^{(2)}(\epsilon) \mathcal{M}_4^{(1)}(2\epsilon) + C^{(2)} + \mathcal{O}(\epsilon) \]

CA, Bern, Dixon, Kosower

\[ \mathcal{M}_n^{(2)}(\epsilon) = \frac{1}{2} \left( \mathcal{M}_n^{(1)}(\epsilon) \right)^2 + f^{(2)}(\epsilon) \mathcal{M}_n^{(1)}(2\epsilon) + C^{(2)} + \mathcal{O}(\epsilon) \]

Bern, Dixon, Smirnov

Can be computed in the strong limit with AdS/CFT

Alday, Maldacena

\[ \ln(1 + \sum_{l=1}^{\infty} a^l M_n^{(l)}) = \ln(1 + \sum_{l=1}^{\infty} a^l W_n^{(l)}) + \mathcal{O}(\epsilon) \]

\[ \mathcal{M}_n = \exp \left[ \sum_{l=1}^{\infty} a^l f^{(l)}(\epsilon) M_n^{(1)}(l\epsilon) + C^{(l)} + \mathcal{O}(\epsilon) \right] \]

\[ <\text{Wilson Loop}> = \text{Amplitude} \]

Sokachev, Korchemsky

Can compute two-loop amplitudes with arbitrary number of legs, using the Wilson-loop duality

CA, Brandhuber, Heslop, Khoze, Spence, Travaglini
Outlook

Our abilities in simulating precisely collider processes have grown tremendously.

New computational methods at NLO are extremely powerful. A classic work which will be part of future field theory books.

Ready to take on the big challenge of finding new physics convincingly in hadron collider data.