String Theory and Supersymmetry

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I will basically be describing two topics:

I) How supersymmetry and string theory grew up together.

III) How string theorists try to derive models of particle physics from string theory:
   a) top-down
   b) bottom-up
   c) hybrid
For our first point, we can basically go back to the origins of supersymmetry, with the Ramond model (1970).

In this period, string theory was studied as a theory of strong interactions, and it had some success, but also some serious problems.

One of them, which Ramond wanted to solve, was that the theory only described bosons.
There is a simple reason for that: the vibrations of a string are bosonic.

Described by a field $X^\mu(\sigma)$ -- here $X^\mu$ are coordinates of space and $\sigma$ measures the position along the string. (Or $X^\mu(\sigma, \tau)$ where $\tau$ is the proper time along the string.) All excitations are bosons.
Ramond’s rather unusual idea was that to get fermions, we usually have gamma matrices $\gamma^\mu$ obeying $\{ \gamma^\mu, \gamma^\mu \} = 2\eta^{\mu\nu}$.

He “promoted” the gamma matrices to fields $\gamma^\mu(\sigma)$ living on a string and obeying a stringy version of the Clifford algebra:

$$\{ \gamma^\mu(\sigma), \gamma^\nu(\sigma') \} = 2\eta^{\mu\nu} \delta(\sigma - \sigma')$$

Then he postulated some clever equations obeyed by $\gamma^\mu(\sigma)$. 
Basically, $\gamma^\mu(\sigma)$ described a new kind of wave traveling on the string.

The new string theory described vibrations of the string, plus the new waves. This made it possible to describe fermions, as well as bosons.
It wasn’t too long before Ramond’s work was reinterpreted. The new field \( \gamma^\mu(\sigma) \) that he had introduced could be better understood by saying that the string didn’t just have one ordinary dimension, labeled by \( \sigma \), but it also has an additional “fermionic” dimension labeled by a coordinate \( \theta \) that is fermionic and (therefore) obeys \( \theta^2 = 0 \) 

“Superfield” 

\[
X^\mu(\sigma, \theta) = X^\mu(\sigma) + \theta \gamma^\mu(\sigma)
\]
Ramond had invented the first supersymmetric model – in the West (There was a parallel origin of supersymmetry in the work of Gol’fand and Likhtman in the USSR.)

It was a model in two dimensions \( \sigma, \tau \)

that is a model on two-dimensional Minkowski spacetime
A conventional model in two dimensional Minkowski spacetime would have for symmetries the translation generators

\[ P_\mu \sim -i \frac{\partial}{\partial x^\mu} \quad \mu = 0, 1 \]

and also the generator \( K \) of a Lorentz boost.
Saying that the model is supersymmetric means that there are additional “fermionic” symmetries – basically coming from the existence of a Dirac-Ramond operator on loop space – that obey

\[ \{ Q_\alpha, Q_\beta \} = \gamma^I_{\alpha\beta} P_I \]

Cute but two-dimensional and so seemingly not relevant to the real world except maybe via string theory.
However, Wess and Zumino in 1974 realized that they could do in four-dimensional spacetime something quite analogous to what Ramond had done on the “worldsheet” of the string.

To the ordinary coordinates $x^\mu$ of spacetime, they added new “fermionic coordinates” $\theta^\alpha$, giving superfields

$$\Phi(x^\mu, \theta^\alpha) = \phi(x^\mu) + \theta^\alpha \psi_\alpha(x^\mu) + \ldots$$

that describe both bosonic and fermionic fields in spacetime, with symmetries between them
For Wess and Zumino, originally there was just an *analogy* between supersymmetry along a string and supersymmetry in spacetime.

But soon it was realized (Gliozzi, Olive, Scherk 1976) that the Ramond (Neveu-Schwarz) model, when properly elaborated, actually has spacetime supersymmetry.
It may sound odd that the same structure of supersymmetry occurs both along a string and (if string theory is correct) in spacetime.

But actually, that is the way string theory usually works.

We don’t really understand it terribly well, but all the most interesting structures have close analogies between what happens on the “string worldsheet” and what happens in “spacetime.”
Classic examples:

• Gauge symmetry
• General covariance, i.e. General Relativity and gravity
• Supersymmetry
• Nonperturbative “dualities” that determine what happens for strong coupling

(I listed these in roughly the order they were discovered.)
So supersymmetry, both along the string and in spacetime, was certainly an interesting new level of structure.

But more than that it was shown (around 1980-2, mostly by Green and Schwarz) that supersymmetry was needed for the theory to make sense.

In fact, despite its partial successes, there had always been internal inconsistencies in string theory … which went away with supersymmetry.
The simplest example of that is that traditionally string theory had a “tachyon” -- the vacuum was unstable.

Supersymmetry eliminated this, since it doesn’t allow tachyons as the Hamiltonian is a sum of squares:

\[ H = \sum_{\alpha} Q_{\alpha}^2 \]

(This is a special case of

\[ \{ Q_{\alpha}, Q_{\beta} \} = \gamma_{\alpha\beta}^I P_I \] )
I should stress that if supersymmetry is relevant to nature, it is “spontaneously broken,” since we do not observe the extra symmetries $\mathcal{Q}_\alpha$ in the real world of elementary particles … for example, there does not exist a boson with the same mass and electric charge as the electron.

So if supersymmetry is relevant to nature, it is like the gauge symmetry of the Standard Model of weak interactions – a symmetry of the microscopic equations but not of the vacuum state.
The LHC – the new accelerator that soon should be operational at CERN – is expected to teach us what has happened to the electroweak gauge symmetry.

If supersymmetry is part of nature and is spontaneously broken only at LHC energies (i.e. around 1 TeV) then the LHC may also discover this.
While we do not really know if supersymmetry is relevant at LHC energies, there are a couple of hints that it might be.

To me, the most striking of these hints have to do with

i) the quantum numbers of quarks and leptons

ii) the value of the weak mixing angle

\[ \sin^2 \theta_W \]
Both of these are suggestive of some sort of “Grand Unification” of the elementary particle forces in which supersymmetry would be part.

But if this is right, we haven’t yet (pre-LHC) reached the energy needed to prove it at accelerators.
By around 1982, it was possible to use string theory to make completely consistent models of quantum gravity interacting with matter (Green, Schwarz, Brink).

The matter included massless and massive bosons and fermions with gauge forces.

But in detail the matter was wrong, so it wasn’t possible to derive models of particle physics from string theory.
The most strikingly wrong feature was that although string-derived models could violate parity, they couldn’t do so in the right way: It wasn’t possible to have the gauge forces violate parity (the approximate symmetry of nature between Left and Right) … as they do in the real world with the V-A structure of weak interactions.
Real model-building only became possible in 1984, when a new anomaly cancellation mechanism (Green and Schwarz) solved the problem of parity violation and the discovery of the heterotic string (Gross, Harvey, Martinec, and Rohm) opened up elegant possibilities for constructing string-based models of particle physics.
By now I’ve finished what I’ll say about my first topic “how supersymmetry and string theory grew up together” and I move on to the second topic, which is to tell a little about models of particle physics derived from string theory.

I’ll start by describing the traditional top-down viewpoint.
First of all, if we are going to make a model of particle physics, we have to start in 10 dimensions … because that is where supersymmetric string theory works.

So spacetime will be $M_4 \times K$

where the first case is four-dimensional Minkowski spacetime and the second factor is some compact six-manifold.
This means that we won’t just be making a theory of quantum gravity – we’ll have to unify gravity with matter, where matter comes from vibrations of the string plus the existence of extra dimensions.

So actually extra dimensions may be a blessing in disguise.
We also essentially know the gauge group in ten dimensions – it is $E_8 \times E_8$ since that is the gauge group that works (for the heterotic string).

The gauge group in four dimensions may be a subgroup of $E_8 \times E_8$ if there is gauge symmetry breaking in the compactification. This is possible, but rather restrictive: the only “Higgs fields” are the higher dimensional part of the gauge fields.
There are a lot of things one might do, but most work has assumed that we are trying to make a model that is supersymmetric at accelerator energies.

Partly this is for reasons that have nothing to do with string theory – the usual arguments for supersymmetry including coupling unification and stabilizing the electroweak mass scale.
But there are also reasons special to string theory:

With supersymmetry, we know what to do to construct approximate vacuum states that are stable at least down to rather low energies ....

If we drop supersymmetry at the unification scale, we will get back the instabilities (think tachyon) that caused trouble before supersymmetry was introduced.

Plus, we’d have too many choices.
Even if we do assume supersymmetry, we have a lot of choices, but some parts of the picture are fairly clear and in fact work nicely.

In the compactification $M_4 \times K$, the second factor $K$ should be a “Calabi-Yau manifold.” There is a similar restriction on the gauge fields that we use to break the ten-dimensional $E_8 \times E_8$ gauge group to a four-dimensional gauge group. (They define a “holomorphic vector bundle.”)
Something nice happens: with the simplest choices of symmetry breaking, the gauge groups that pop out in four dimensions are the usual GUT groups

$$SU(5), \ SO(10), \ E_6$$

(and their Standard Model – like subgroups at lower energies)

together with the usual GUT representations that contain quarks and leptons. (Here GUT’s are “Grand Unified Theories” of particle interactions.)
Many things come out nicely in such constructions,
but we know little about just what model to pick and what mechanism of supersymmetry breaking dominates.

I already stressed that supersymmetry breaking is every bit as important as supersymmetry, since the real world as we see it is not supersymmetric at ordinary energies.
Not knowing what Calabi-Yau manifold to pick or what is the right approach to supersymmetry breaking makes it pretty hard to get past a rough draft of the real world – by a rough draft I mean the forces and the quantum numbers of particles – but there is incremental progress, for instance see

Dundee, Raby, and Wingert, arXiv:0805.4186
What I have described so far is the top-down approach to string theory model-building. One aims to embed the Standard Model in a completely consistent unified theory of quantum gravity plus matter. It is inevitable to try to do this once it becomes possible to try.

Even if the problem hasn’t been completely solved, it is a real accomplishment to have a class of constructions that come close and probably include examples that do work.
What I’ve described so far is the top-down approach to string theory model-building. There is also a more “bottom-up” approach that has been developed since new tools of the 1990’s – such as the theory of “branes” – made this possible.

Here one observes that a full string construction, even if it does describe the Standard Model, will inevitably describe all kinds of other stuff besides.
Perhaps it is not necessary to have a fully elaborated string compactification in order to describe the Standard Model.

Maybe the part that we are able to observe at accelerators depends only on part of the full string theory construction. Certainly it would be attractive to be able to describe observable physics without all kinds of hypotheses about a complete model.
This type of thinking leads to the idea of localizing the Standard Model on a region of spacetime, such as a brane $B$

supported on a submanifold of the compact manifold $K$
A variant is that the Standard Model might “live” near a singularity of $K$.

$K$ might be very complicated but the details might not matter very much if Standard Model fields live near the singularity.
This type of thinking is also related to other models that don’t necessarily require string theory though it is possible to try to embed them in string theory…

Models with large extra dimensions or a “low” quantum gravity scale, possibly even accessible to the LHC.
Bottom-up models have some clear advantages. Once might be able to understand accelerator physics without having to understand “everything”; and they’ve led to a lot of fresh thinking.

At the same time, most bottom-up models have a clear disadvantage – they treat the apparent unification of the strong, weak, and electromagnetic couplings as an accident.
Certainly the apparent coupling unification may be an accident, but that seems like a pity.

So I want to conclude by mentioning very recent work that tries to combine the virtues of the top-down and bottom-up approaches.

The Standard Model is supported on a submanifold

But now this submanifold is complicated enough to support a GUT-like construction, leading to coupling unification.
At a minimum this is an interesting new “twist.” Part of what makes it interesting is that it is highly constrained – it just barely works.

Because it is highly constrained, it has led to interesting new perspectives on many questions such as the CKM mixing, neutrino masses, and mechanisms of supersymmetry breaking.
But we really don’t know enough to get it right.

We need to know a lot more, and most fundamentally we need to know from the LHC whether the idea of supersymmetry at accelerator energies is correct.