The inclusive determination of $V_{ub}$

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The Unitarity Triangle

\[ L_W = -\frac{g}{2\sqrt{2}} V_{ij} \bar{u}_i \gamma^\mu W^\mu_+ (1 - \gamma^5) d_j + \text{h.c.} \]

Unitarity determines several triangles in complex plane

\[ V_{ij} V_{jk}^* = \delta_{ik} \]

\[ V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \]

\[ 1 + \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} + \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} = 0 \]

\( O(\lambda^3) \)

\[ |V_{ub}/V_{cb}| \text{ describes a circle in the } (\rho, \eta) \text{ plane} \]

\( V_{td} \) cannot be accessed directly: we resort to loop transitions

FCNC sensitive to new physics

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Only the CKM?

$\bar{\rho} = 0.147 \pm 0.029$

$\bar{\eta} = 0.342 \pm 0.016$

Uncertainties mostly theoretical!

Precision flavor physics can progress only together with our understanding of QCD!

Inclusive B decays are interesting because well measured and accurately predicted.
The present situation

• UTfit (indirect): \(|V_{ub}| = (3.44 \pm 0.16) \times 10^{-3}\)

• Exclusive determination: \(|V_{ub}| = (3.4 \pm 0.4) \times 10^{-3}\)

• Inclusive determinations, HFAG averages:
  \(|V_{ub}| = (4.31 \pm 0.17 \pm 0.35) \times 10^{-3}\) (BLNP)
  \(|V_{ub}| = (4.34 \pm 0.16 \pm 0.25) \times 10^{-3}\) (DGE)
  \(|V_{ub}| = (3.98 \pm 0.15 \pm 0.30) \times 10^{-3}\) (BLNP no bsγ)

\(|V_{ub}| = (3.98 \pm 0.15 -0.35+0.25) \times 10^{-3}\) NEW
The problem seems to be in the Inclusive determination…
A set of interdependent measurements

| Process     | Type  | BR | |Vcbl| |Vubl|  |
|-------------|-------|----|---|----------------|--|---|
| b→c l ν     | tree  | BR~10% | |Vcbl| |
| b→u l ν     | tree  | ~10^{-3} | |Vubl| |
| b→s γ       | loop  | ~3 10^{-4} | new physics, |Vtsl| |
| b→d γ       | loop  | ~ 10^{-6} | new physics, |Vtdl| |
| b→ sl^+l^-  | loop  | ~6 10^{-6} | new physics |

Not only BR are relevant: various asymmetries, spectra etc
What do they have in common?

Simplicity: ew or em currents probe the B dynamics

<table>
<thead>
<tr>
<th>INCLUSIVE</th>
<th>EXCLUSIVE</th>
</tr>
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<tbody>
<tr>
<td><strong>OPE:</strong> non-pert physics described by B matrix elements of local operators can be extracted by exp suppressed by $1/m_b^2$</td>
<td><strong>Form factors:</strong> in general computed by non pert methods (lattice, sum rules,...) symmetry can provide normalization</td>
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B factory accuracy challenges our theoretical understanding
The advantage of being inclusive

The decay of a B meson is not the decay of a free b quark!

However $\Lambda_{QCD} \ll m_b$: inclusive decays via OPE admit systematic expansion in $\alpha_s$ and $\Lambda_{QCD}/m_b$ starting with $\Lambda_{QCD}^2 / m_b^2$.

- Leading term is parton model.
- Non-pert corrections are generally small and under control.
A double expansion

Differential rate can be related to $\text{Im}$ of

$$h_{\mu\nu}(q^2, q_0) = \frac{1}{2M_B} \langle B | \int d^4x \ e^{-iqx} iT \left\{ J_{\mu}(x), J_{\nu}^+(0) \right\} | B \rangle$$

OPE (HQE): $T J(x) J(0) \approx c_1 \bar{b}b + c_2 \bar{b} \vec{D}^2 b + c_3 \bar{b}\sigma \cdot Gb + \ldots$

- The leading term is parton model, $c_i$ are series in $\alpha_s$
- New operators have non-vanishing expectation values in $B$ and are suppressed by powers of the energy released, $E_r \sim m_b$
- No $1/m_b$ correction!

Non-pert parameters: $m_b, m_c$, $1/m_b^2$, $\mu_G^2, \mu_\pi^2$, $1/m_b^3$, $\rho_D^3, \rho_{LS}^3$

These parameters are now known with reasonable accuracy from Inclusive semileptonic $B$ decays

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Leptonic moments:

\[
\langle E_{\ell}^n \rangle_{E_{\text{cut}}} = \frac{\int_{E_{\text{cut}}} dE_{\ell} \; E_{\ell}^n \frac{d\Gamma}{dE_{\ell}}}{\int_{E_{\text{cut}}} dE_{\ell} \frac{d\Gamma}{dE_{\ell}}} 
\]
Hadronic mass moments

NEW
December 2007
(new code)
P. Giordano, PG

Fit in the kinetic scheme
Global fit to $|V_{cb}|$, $BR_{s\bar{t}}$, HQE

Buchmuller & Flacher 06

Result of fit to all moment measurements:

$|V_{cb}|@ 2%$

$m_b < 1%$

$m_c @ 5%$

In $\overline{\text{MS}}$ scheme:

$m_b(m_b) = 4.20 \pm 0.04 \text{ GeV}$

$m_c(m_c) = 1.24 \pm 0.07 \text{ GeV}$

$m_c(\mu)/m_b(\mu) = 0.235 \pm 0.012$

Based on Gambino & Uraltsev, Benson et al

Good agreement with other similar analyses:

Bauer et al. hep-ph/0408002

DELPHI hep-ex/0510024

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**Testing parton-hadron duality**

- **What is it?** For all practical purposes: the OPE. No OPE, no duality.

- **Do we expect violations?** Yes, probably. OPE must be continued analytically. There are effects that cannot be described by the OPE, like hadronic thresholds. Expected small in semileptonic decays.

- **Can we constrain them effectively?** In a self-consistent way: just check the OPE predictions. E.g. leptonic vs hadronic moments. Models may also give hints of how it works.

- **Caveats?** HQE depends on many parameters and we know only a few terms of the double expansion in $\alpha_s$ and $\Lambda/m_b$. 
\[ |V_{ub}| \text{ (not so much) inclusive} \]

\[ |V_{ub}| \text{ from total BR}(b \to u\ell\nu) \text{ almost exactly like incl } |V_{cb}| \text{ but we need kinematic cuts to avoid the } \sim 100x \text{ larger } b \to c\ell\nu \text{ background:} \]

\[ m_X < M_D \quad E_i > (M_B^2 - M_D^2)/2M_B \quad q^2 > (M_B - M_D)^2 \ldots \]

or combined \((m_X, q^2)\) cuts

The cuts destroy convergence of the OPE, supposed to work only away from pert singularities.

Rate becomes sensitive to “local” b-quark wave function properties like Fermi motion

\[ \Rightarrow \text{ at leading in } 1/m_b \text{ SHAPE FUNCTION } f(k^+) \]

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Cutting on the hadronic invariant mass spectrum gives more rate, but has the same problem with fermi motion

(Falk, Ligići, Wise, Dikeman, Uraltsev)
Each strategy has pros and cons

<table>
<thead>
<tr>
<th>cut</th>
<th>% of rate</th>
<th>good</th>
<th>bad</th>
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</table>
| $E_L > \frac{m_B^2 - m_H^2}{2m_B}$ | $\sim 10\%$ | don't need neutrino | depends on $f(k^+)$ (and subleading corrections)  
- WA effects largest  
- reduced phase space - duality issues! |
| $s_H < m_H^2$ | $\sim 80\%$ | lots of rate | depends on $f(k^+)$ (and subleading corrections)  
- need shape function over large region |
| $q^2 > (m_B - m_D)^2$ | $\sim 20\%$ | insensitive to $f(k^+)$ | very sensitive to $m_b$  
- WA corrections may be substantial  
- effective expansion parameter is $1/m_C$ |
| “Optimized cut” | $\sim 45\%$ | - insensitive to $f(k^+)$  
- lots of rate  
- can move cuts away from kinematic limits and still get small uncertainties | sensitive to $m_b$ (need +/- 60 MeV for 5% error in best case) |
| $P_+ > \frac{m_D^2}{m_B}$ | $\sim 70\%$ | - lots of rate  
- theoretically simplest relation to $b \rightarrow s\gamma$ | depends on $f(k^+)$ (and subleading corrections) |
Totally inclusive measurements impossible: needs cuts that **destroy** local OPE, so successful in $b \rightarrow c \ell \nu$. Still non-pert contributions can be resummed into distribution functions (Fermi motion).

Different cuts lead to similar $V_{ub}$. But are present theoretical errors realistic?

How much can they be reduced in the future?
The photon–energy spectrum in perturbation theory

Perturbation theory is badly divergent: Sudakov double logs near the endpoint; huge corrections
Resummed perturbation theory is qualitatively different: **Support properties; stability!**

Power corrections are small: resummed perturbation theory yields a good approximation to the meson decay spectrum.

**b quark SF emerges from resummed pQCD but needs an IR prescription and power corrections for \(b \to B\)**

Aglietti et al use analytic coupling.
SF approaches: I. DGE

**Dressed Gluon Exponentiation**

**Renormalon resummation:**
summation of PT to power accuracy, giving access to NP power corrections. Renormalons $\equiv n \to \infty$ asymptotics,

$$\int_0 dk^2 \left[ \alpha_s \beta_0 \ln(k^2/Q^2) \right]^n \Rightarrow \left[ \alpha_s \beta_0 \right]^n n!$$

**Sudakov resummation:**
corrections that become large near the exclusive phase–space boundary, $m_{\text{jet}} \to 0$

$$\left[ C_F \alpha_s \ln^2 \left( m_{\text{jet}}^2 / Q^2 \right) \right]^n$$

**Dressing the gluon:** $\alpha_s$ correction in moment space

**Dressed Gluon Exponentiation:**
Sudakov resummation to power accuracy, giving access to non-perturbative power corrections that become large near the exclusive phase–space boundary.

DGE amounts to genuine definition of Fermi motion based on dynamics of on-shell $b$ decay + power suppr. corrs
The results of different cuts are all consistent.

\[ \chi^2 / \text{dof} = 2.3 / 6 \quad (CL = 89\%) \]

Smallest uncertainty: \( |V_{ub}| = (4.34 \pm 0.16 \pm 0.25) \times 10^{-3} \)

Would average \( m_b \) is used,

\[ m_b^{\text{MS}}(m_b) = 4.20 \pm 0.07 \text{ GeV} \]

\( m_b \): the largest source of error.

Relies on PT to compute the SF!

Additional (missing) power corrections can be included, but relation to local OPE needs to be clarified.
Shape function (II)

The local OPE breaks down close to boundaries. In the threshold region new dynamics emerges.

Expanding a shape of width $\sim \Lambda$ in powers of $\Lambda/m_b$ one gets increasingly singular terms

$$\frac{d\Gamma}{dE_\gamma} \propto \delta\left(E_\gamma - \frac{m_b}{2}\right) + \frac{\Lambda^2}{m_b^2} \left[ \delta\left(E_\gamma - \frac{m_b}{2}\right), \delta''\left(E_\gamma - \frac{m_b}{2}\right) \right] + \ldots$$

Describing the shape requires summing most singular terms into a function of a single variable

$$f(k_+) = \langle \delta(iD_+ - k_+) \rangle = \frac{\langle B(v) | h_v, \delta(iD_+ - k_+) | h_v | B(v) \rangle}{\langle B(v) | h_v, h_v | B(v) \rangle}$$

First few moments of $f(k)$ fixed by local OPE

$$\int dk_+ k_+^n f(k_+)$$

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SF approaches: II. Multiscale OPE

Neubert et al

\[ d\Gamma = H J \otimes \hat{S} + \frac{1}{m_b} H_i J_i \otimes \hat{S}_i + \ldots \]

Energy scale separation (Factorization)

Many largely unconstrained Subleading SFs

\[ \sim \Delta = m_b - 2 E_{\text{cut}} \]

SF is parameterized perturbative resummation automatic

O. Lange (MIT)
Here $m_b$ is based on the combined fit of $\bar{B} \rightarrow X_c l \bar{\nu}$ and $\bar{B} \rightarrow X_s \gamma$ data. Present preference is to discard $\bar{B} \rightarrow X_s \gamma$ data leading to a 77 MeV (1.3 $\sigma$) increase in $m_b$, and a corresponding decrease in $|V_{ub}|$:

$$4.31 \cdot 10^{-3} \rightarrow 3.98 \cdot 10^{-3}$$

Note however that the older (combined) $m_b$ value is better consistent with the world average $m_b$, and with a very precise recent determination from $e^+ e^- \rightarrow$ hadrons by Kühn et al.
A new “prudent” theoretical analysis

- **kinetic scheme.** Wilsonian infrared cutoff $\mu \sim 1 \text{ GeV}$: contribution of soft gluons absorbed into definition of OPE parameters AND distribution function(s)
- **Fermi motion:** finite $m_b$ SF, includes all available subleading corrections
- local OPE breaks down at **high $q^2$**: need to model the tail, consistent with positivity, **WA** naturally emerge.
- Triple differential distribution including all known pert and nonpert effects, c++ code designed as generator

Perturbative corrections

In the kin scheme soft gluon emission is inhibited
the spectrum has only collinear singularities

- no resummation of collinear logs
  (unnecessary)

- complete implementation of
  $O(\alpha_s^2 \beta_0)$ corrections $\Rightarrow$ -5% in $V_{ub}$

PG, Gardi, Ridolfi
**Fermi motion (I)**

**Leading SF** resums leading twist effects, $m_b \to \infty$ universal, $q^2$ indep

**Finite $m_b$ distribution functions** include all $1/m_b$ effects, non-universal no need for subleading SFs

\[
F(k_+) \rightarrow F_i(k_+, q^2, \mu)
\]

Structure function $(i = 1, 2, 3)$

$q^2$ dependence
cutoff dependence (gluons with $E_g < \mu$)

\[
\frac{d^3 \Gamma}{dq^2 dq_0 dE_\ell} = \frac{G_F^2 |V_{ub}|^2}{8\pi^3} \left\{ q^2 W_1 - \left[ 2 E_\ell^2 - 2q_0 E_\ell + \frac{q^2}{2} \right] W_2 + q^2 (2E_\ell - q_0) W_3 \right\}
\]

\[
W_i(q_0, q^2) = m_b^{n_i}(\mu) \int dk_+ F_i(k_+, q^2, \mu) \, W_i^{pert} \left[ q_0 - \frac{k_+}{2} \left( 1 - \frac{q^2}{m_b M_B} \right), q^2, \mu \right]
\]

This factorization formula perturbatively defines the distribution functions

see also Benson, Bigi, Uraltsev for bsv

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Fermi motion (II)

\[ \int dk_+ k_+^n F_i(k_+, q^2) = \text{local OPE prediction} \]

The variance gets negative

Importance of subleading effects
The high $q^2$ tail

Higher dimensional operators are not suppressed at high $q^2$ leading to pathological features. Origin in the non-analytic square root

$$\frac{d\Gamma}{dq_0 \, dq^2} \propto \sqrt{q_0^2 - q^2}$$

$$\frac{d\Gamma}{dq^2} \sim -\sum_{n=1}^{\infty} \frac{(-1)^n \, b_n(q^2)}{(1 - \hat{q}^2)^{n-2}} \left( \frac{\Lambda}{m_b} \right)^n$$

In the integrated rate the $1/m_b^3$ singularity is removed by the WA operator: needs modelling for $q^2$ spectrum

$$\delta \Gamma \sim \left[ C'_{WA} \, B_{WA}(\mu_{WA}) - \left( 8 \ln \frac{m_b^2}{\mu_{WA}^2} - \frac{77}{6} \right) \frac{\rho_D^3}{m_b^3} + \mathcal{O}(\alpha_s) \right]$$

WA matrix element $B_{WA}$ parameterizes global properties of the tail
High $q^2$ (II)

Two models of the high $q^2$ tail $q^2 > q^*^2$ plus a $\delta(q^2 - m_b^2)$ for the rest of WA $8.5 < q^*^2 < 13.5$ GeV$^2$

NB: O(1) mixing between WA and Darwin ops. The isosinglet part of WA can be substantial. Tail analysis is complementary to $B^+/B_d$.
Functional forms

About 100 forms considered, large variety. Small uncertainty (1-2%) on $V_{ub}$. 

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Theoretical errors

- Parametric errors generally dominant, in particular $m_b$, 3-4%
- Perturbative corrections 2-3%
- Functional form 1-2%
- Modelling of the $q^2$ tail and WA depending on cut from 0 to 7%. WA tends to decrease $V_{ub}$

Overall theory errors are 5-9%, depending on the cuts.
This employs the HFAG average for $m_b$ etc (bsy included)

Preliminary HFAG Average

$|V_{ub}| = (3.98 \pm 0.15^{+0.25}_{-0.35}) \times 10^{-3}$
Constraining Weak Annihilations

Preliminary Babar analysis of the $q^2$ spectrum seems to suggest a small WA contribution and $V_{ub} \approx 0.0040$
$V_{ub}$ determinations with $B_{WA}(1\text{GeV})=0.0188$

All determinations consistent with UTfit
Summary

✓ New, relatively simple method available, theory errors investigated in detail

✓ high $q^2$ determination has larger errors, WA tend to decrease $V_{ub}$

✓ possibility to constrain WA using $q^2$ spectrum + $B^+/B^0$

✓ $V_{ub}$ best determination at present is $M_X$ cut, compatible with exclusive and marginally with indirect (fit) determinations. Further improvements will come from an upper cut on $q^2$. 
Prospects

• Improvements from knowing the b mass and the OPE pars more precisely lattice, moments, pert calculations, goal 20 MeV for $m_b$

• from studying all the spectra of $b \rightarrow u \ell \nu$ to constrain WA and the SFs

• from more inclusive measurements to minimize dependence on functional forms AND $m_b$

Learning from data
a 2% error at super B might be possible
Main theoretical desiderata

- know the b mass and the OPE pars precisely lattice, b→c and bsqγ moments, pert calculations, goal 20 MeV for \( m_b \)
- study all the spectra of b→u l ν to constrain WA and the SFs, complementary to OPE constraints
- be as inclusive as possible to minimize dependence on functional forms

Present parametric is 3.5% with \( \delta m_b \sim 40 \) MeV, dominates cleanest cuts
An almost inclusive measurement is less sensitive to \( m_b \).
From b→c experience, duality violation should be small
Therefore, a 2% goal on \(|V_{ub}|\) seems to be realistic.
the Future

- Non-linear approach to selecting signal events may be used to improve space coverage with >400 fb⁻¹? samples

| $|V_{ub}|$ measurement (using BLNP) | $f_u$  | uncertainties (%) |
|----------------------------------|-------|--------------------|
|                                  |       | stat   | sys   | the   | tot   |
| PLB 486:86 BaBar: $M_x<2.5\text{GeV}$ | ~90%  | 18.2   | 7.8   | 2.6   | ~20   |
| PRL 96 (2006) 221801 Belle: $M_x<1.7\text{GeV}$ | ~47%  | 5.0    | 4.8   | 5.9   | ~9    |
| arxiv/0708.3702 BaBar: $M_x<1.55\text{GeV}$ | ~41%  | 3.7    | 3.0   | 7.0   | ~9    |
| where can we go now?             | ~80-90% | ?     | ~4    | ~2.6  | ?     |

- If almost fully inclusive: Shape Function not needed!
- $|V_{ub}|$ theory errors are highly correlated.
- Measurements of the full phase space are crucial to reducing the error on $|V_{ub}|$ in the world average!
There is NO normalization of form f.s from HQ symmetry

New first unquenched results
lattice errors still ~11-15%

Sum rules good at low $q^2$
lattice at high $q^2$: complement each other

$q^2$ extrapolation from theory
bounds plus data: FF normalization at 1 point is sufficient
Ball-Zwicky, Becher-Hill etc

Lattice (distant) goal is 5-6%

New strategy using combination
of rare B,D decays Grinstein & Pirjol
<table>
<thead>
<tr>
<th>FF calculation</th>
<th>$V_{ub} \times 10^{-3}$</th>
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<tbody>
<tr>
<td>Ball-Zwicky $q^2 &lt; 16$</td>
<td>$3.36 \pm 0.15 +0.55-0.37$</td>
</tr>
<tr>
<td>HPQCD $q^2 &gt; 16$</td>
<td>$4.20 \pm 0.29 +0.63-0.43$</td>
</tr>
<tr>
<td>FNAL $q^2 &gt; 16$</td>
<td>$3.75 \pm 0.26 +0.65-0.43$</td>
</tr>
<tr>
<td>APE $q^2 &gt; 16$</td>
<td>$3.78 \pm 0.26 +1.45-0.67$</td>
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**| $V_{ub}$ from $B \rightarrow \pi l \nu$**

**full range**

| $\Gamma(q^2 > 16 \text{GeV}^2)/|V_{ub}|^2$ |
|--------------------------------|
| UKQCD                          |
| APE                            |
| FNAL                           |
| JLQCD                          |
| FNAL/MILC                      |
| HPQCD                          |

$N_f=0$

$N_f=2+1$

**Unquenched results probably not yet mature:** handle with care
Global fit (kinetic scheme)

New

with BaBar’s
Breco $B \rightarrow X_s \gamma$ and $M^H_n$

Babar $M^{\ell}_{1,2} \times 3$ (1.9–2.0 GeV)
Babar $M^H_{1,2}$ (0.9–1.5 GeV(??))
Babar $M^{\ell}_{0,1,2,3}$ (0.6–1.5 GeV)
Belle $M^{\ell}_{1,2}$ (1.8–2.0 GeV)
Belle $M^H_{1,2}$ (0.7–1.3 GeV)
Belle $M^{\ell}_{0,1,2,3}$ (0.6–1.4 GeV)
CLEO $M^{\ell}_{1}$ (2.0 GeV)
CLEO $M^H_{1,2}$ (1.0–1.5 GeV)
DELPHI $M^H_{1,2,3}$ (0 GeV)
DELPHI $M^{\ell}_{1,2,3}$ (0 GeV)
CDF $M^H_{1,2}$ (0.7 GeV)
old HFAG $B$ (0.6 GeV)

$|V_{cb}| = (42.04 \pm 0.34_{\text{fit}} \pm 0.59_{\text{G}}) \times 10^{-3}$

$m_b^{\text{kinetic}} = 4.597 \pm 0.034_{\text{fit}}$ GeV

$m_c = 1.1634 \pm 0.051_{\text{fit}}$ GeV

$\mu^2_{\pi} = 0.4341 \pm 0.033_{\text{fit}}$ GeV$^2$

$\rho_D^3 = 0.2927 \pm 0.020_{\text{fit}}$ GeV$^2$

$\Delta \chi^2 = 2.3$ contour, 68% CL

$b \rightarrow c \ell \nu + b \rightarrow s \gamma$

$b \rightarrow c \ell 
u$

$b \rightarrow s \gamma$

(68% & 39% CL)
The disadvantage of cuts

The local OPE breaks down close to boundaries: unreliable unless cut is low enough.

In the endpoint region $E_\gamma \sim 2$ GeV new dynamics: distribution function (Shape Function=SF) and Sudakov resummation.

\[ f(k_+) = \langle \delta(iD_+ - k_+) \rangle = \frac{\langle B(v) | \tilde{h}_v \delta(iD_+ - k_+) h_v | B(v) \rangle}{\langle B(v) | h_v h_v | B(v) \rangle} \]

For $m_b \to \infty$

First few moments of $f(k)$ fixed by local OPE

OPE certainly valid for $E_{cut} \sim 1\text{-}1.2$ GeV. Experiments cut at 1.8\text{-}1.9 GeV. What happens in between?

Extrapolation to 1.6 GeV is now performed by HFAG using $b \to c\bar{l}v$ input $\Rightarrow$ small 2% error counted in the *experimental* budget
Strong dependence on the value of $m_b$ known with about 40-50 MeV uncertainty.

Experimental situation far from settled!

**CLEO ($E_\gamma$)**
3.52 ± 0.41 ± 0.35

**BELLE sim. ann. ($m_X, q^2$)**
3.97 ± 0.42 ± 0.31

**BELLE ($E_\gamma$)**
4.35 ± 0.40 ± 0.33

**BABAR ($E_\gamma$)**
3.89 ± 0.22 ± 0.33

**BABAR ($E_\gamma, s_{b}^{max}$)**
3.94 ± 0.27 ± 0.39

**BELLE ($m_X$)**
3.66 ± 0.24 ± 0.27

**BABAR ($m_X$)**
3.74 ± 0.18 ± 0.31

Average +/- exp +/- (mb, theory)
3.98 ± 0.15 ± 0.30

$\chi^2$/dof = 6.3/6 (CL = 39 %)

OPE-HQET-SCET (BLNP)

Phys.Rev.D72.073006,2005

$m_b$ input from $b \to c l v$ moments

$|V_{ub}|$ $[\times 10^{-3}]$ 6

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Weak annihilation

\[ C_D \simeq -\frac{\Gamma_0}{m_b^3} \left[ 8 \ln \frac{m_b^2}{m_q^2} - \frac{77}{6} + O\left(\frac{m_q^2}{m_b^2}\right) \right]. \]

coefficient of Darwin operator

\[ \delta\Gamma_{WA} = \Gamma_0 C_{WA} \langle B|O_{WA}^u|B\rangle \]

adding 1loop corrections

\[ \Sigma_q \]

BAD: WA $\lesssim 3\%$ in rate but gets enhanced in phase space corners

GOOD: WA small but can be experimentally constrained, not only in $B^+/B_0$

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