An Effective Theory of Quintessence Perturbations

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Effective Theory

Outline

- Effective Theory
- The Quintessence Plane

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- Limits and Observations

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- Limits and Observations
- Conclusions and Future Work

The general idea


Usual approach to (inflation and) DE models:

1. Take a Lagrangian for a scalar field
2. Solve EOM for scalar + FRW equations, to find an accelerating expansion
3. Study perturbations around this solution
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We can construct a stable model violating the NEC \((w < -1)\)

Difference with inflation

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- We are interested in the sub-horizon dynamics
Constructing the action

Start by choosing gauge comoving with scalar field: $\delta \phi(\vec{x}, t) = 0$. Time diffs are broken: spatial diffs remain. Then the most generic Lagrangian will contain:

- Generic functions of time
- $\partial_\mu t = \delta_\mu^0 \implies$ upper 0 indices are ok: $g^{00}$, $R^{00}$
- Geometric objects of the 3d spatial slices: e.g. extrinsic curvature

$$K^\mu_\nu = H \delta^\mu_\nu + \delta K^\mu_\nu$$

$$S = \int d^3x \, d^3t \sqrt{-g} \left[ F(R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_\mu, t) \right]$$
Expanding up to second order:

\[ S = \int d^3 x \, dt \sqrt{-g} \left[ \frac{M^2_{\text{Pl}}}{2} R + L_m + c(t) g^{00} - \Lambda(t) + \ight. \\
+ \frac{M^4(t)}{2} (g^{00} + 1)^2 - \frac{\bar{M}^2(t)}{2} \delta K^2 \\
- \frac{\bar{M}^2(t)}{2} \delta K^i \delta K^j - \frac{\hat{M}^3(t)}{2} \delta K (g^{00} + 1) \right] \\

Fixing the tadpoles:

\[ \rho_Q = T^{(Q)}_{00} = \Lambda(t) - c(t) \]

\[ p_Q = \frac{1}{3} g^{ij} T^{(Q)}_{ij} = -[c(t) + \Lambda(t)] \]
Finally:

\[ S = \int d^3 x \, dt \, \sqrt{-g} \left[ \frac{M_{\text{Pl}}}{2} R + L_m \right] \]
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- Quintessence
Finally:

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- Quintessence
- k-Essence
Finally:

\[ S = \int d^3x \, dt \, \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R + L_m + \rho_Q - \frac{1}{2} (\rho_Q + p_Q)(g^{00} + 1) + \frac{M^4}{2} (g^{00} + 1)^2 - \frac{\bar{M}^2}{2} \delta K^2 \right] \]

- Quintessence
- k-Essence
- Ghost Condensate
Fluctuation Lagrangian

We restore full diff invariance by performing a (broken) time diff $t \rightarrow t + \xi^0$, and promoting $\xi^0(x) = -\pi(x)$.

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$t \rightarrow t + \xi^0$, and promoting $\xi^0(x) = -\pi(x)$.
Then we choose synchronous gauge:

$$S = \frac{1}{2} \int d^3x dt a^3 \left[ 4M^4 \dot{\pi}^2 + (\rho_Q + p_Q)\pi^2 - (\rho_Q + p_Q) \frac{(\nabla \pi)^2}{a^2} + 
+ 3H(\rho_Q + p_Q)\pi^2 - (\rho_Q + p_Q)\dot{\pi} - \frac{\bar{M}^2}{2} \left( \frac{\dot{h}}{2} - \frac{\nabla^2 \pi}{a^2} \right)^2 \right]$$
We can now study the theoretical constraints on quintessence
\[ S \supset \frac{1}{2} \int d^3x \, dt \, a^3 \left[ 4M^4 \dot{\pi}^2 + (\rho_Q + p_Q) \dot{\pi}^2 - (\rho_Q + p_Q) \frac{(\nabla \pi)^2}{a^2} \right] \]

No ghosts: Classical and quantum stability $\implies 4M^4 > \rho_Q + p_Q$

$c_s^2$ has the same sign as $w + 1$
\[
S \supset \frac{1}{2} \int d^3x \, dt \, a^3 \left[ 4M^4 \dot{\pi}^2 + (\rho_Q + p_Q) \dot{\pi}^2 - (\rho_Q + p_Q) \frac{(\nabla \pi)^2}{a^2} \right]
\]

\[c_s^2 > 1 \iff M^4 < 0\] implies a non Lorentz-invariant UV completion

Arkani-Hamed et al., hep-th/0602178
\[
S \gtrsim \frac{1}{2} \int d^3x \, dt \, a^3 \left[ - (\rho_Q + p_Q) \frac{(\nabla \pi)^2}{a^2} - \bar{M}^2 \left( \frac{\nabla^2 \pi}{a^2} \right)^2 \right]
\]

Cosmological modes, \( \frac{k}{a} \sim H \), have \( \omega \sim c_s^2 k \) for \( |(1 + w_Q)\Omega_Q| \gg \frac{\bar{M}^2}{M_{Pl}^2} \)
\[ S \supset \frac{1}{2} \int d^3x \, dt \, a^3 \left[ 4M^4 \pi^2 - (\rho_Q + p_Q) \frac{(\nabla \pi)^2}{a^2} - \bar{M}^2 \left( \frac{\nabla^2 \pi}{a^2} \right)^2 \right] \]

Now, \( \omega_{\text{inst}} \simeq (1 + w_Q)\Omega_Q \frac{M_{\text{Pl}}^2 H^2}{M^2 \bar{M}} \); \( \omega_{\text{inst}} \ll H \implies c_s^2 \ll \frac{H \bar{M}}{M^2} \)

\( M > (1\text{mm})^{-1} \) is the cutoff of my theory: \(|c_s^2| < 10^{-30}! \)
\[ S = \frac{1}{2} \int d^3x \, dt \, a^3 \left[ 4M^4 \dot{\pi}^2 + (\rho_Q + p_Q) \dot{\pi}^2 - (\rho_Q + p_Q) \left( \frac{\nabla \pi}{a^2} \right)^2 + 3 \dot{H} (\rho_Q + p_Q) \pi^2 - (\rho_Q + p_Q) \dot{h} \pi - \frac{\bar{M}^2}{2} \left( \frac{\dot{h}}{2} - \frac{\nabla^2 \pi}{a^2} \right)^2 \right] \]

Nothing strange happens when you cross \( w = -1 \)

The phantom divide is really... a phantom!
$\omega < -1$: k-essence limit

Neglect 4-derivative term:

$$|1 + w_Q| \Omega_Q \gg \frac{M^2}{M_{Pl}^2} \implies \omega \simeq c_s k \text{ with } c_s \simeq 0$$
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Quintessence escapes (!) from DM potential wells: $\delta_Q \simeq \frac{1+w}{1-3w} \delta_m$
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Is it possible to experimentally distinguish $c_s = 0$ from $c_s = 1$?

Until which value of $w + 1$?
ISW-Galaxy correlation

Distinction possible if $|1 + w_Q| > 0.05$?

Corasaniti, Giannantonio, Melchiorri, 2005

Hu, Scranton, 2004
**$w \to -1$: Cosmological constant limit**

For cosmo scales $|1 + w_Q| \Omega_Q \ll \frac{\bar{M}^2}{M_{Pl}^2}$, $\omega \approx k^2$

**Ghost-condensate limit:**

$$S = \frac{1}{2} \int d^3 x \int d t a^3 \left[ 4 M^4 \dot{\pi}^2 - \bar{M}^2 \left( \frac{\dot{h}}{2} - \frac{\nabla^2 \pi}{a^2} \right)^2 \right]$$

Modification of gravity at very small scales, but in a very long time!
The scalar degree of freedom does not disappear even if $w = -1$

$$\ddot{\pi} + 3H \dot{\pi} = -\frac{\bar{M}^2}{12 M^4 M_{Pl}^2} \nabla^2 \delta \rho_m$$

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\[
\delta \rho_Q = 4M^4 \dot{\pi} \sim \frac{M^2}{M_{Pl}^2} \delta \rho_m \ll \delta \rho_m \quad \text{No relevant perturbations!}
\]
The $\hat{M}$ operator

So far, we neglected the term $\sqrt{-g}M^3g^{00}\delta K \to a^3\dot{M}^3 \left[ \frac{\dot{h}}{2}\dot{\pi} + \dot{\pi}\frac{\nabla^2\pi}{a^2} \right]

In Minkowski background it is a total derivative
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In FRW it is a correction to the spatial kinetic term:

$$S \supset \frac{1}{2} \int d^4 x a^3 \left[ (\rho_Q + p_Q + 4M^4) \dot{\pi}^2 - \left( \rho_Q + p_Q + H\hat{M}^3 \right) \frac{\nabla \pi^2}{a^2} \right]$$

However, in the GC limit it can be suppressed by shift symmetry
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Main effect: if $H\hat{M}^3 > \rho_Q + p_Q$, no gradient instability at all!

However, still have $c_s^2 = \frac{\rho_Q + p_Q + H\hat{M}^3}{\rho_Q + p_Q + 4M^4} \sim \frac{H}{\hat{M}} \sim 0$
Conclusions and future work

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- Understand the possible observational consequences
Thanks!